2 Three assessment tasks and five sample responses on each

Text Messaging

1. How many text messages are sent if four people all send messages to each other?
2. How many text messages are sent with different numbers of people?
3. Approximately how many text messages would travel in cyberspace if everyone in your school took part?
4. Can you think of other situations that would give rise to the same mathematical relationship?

This was adapted from Sending texts – *a task from the Nuffield Foundation's* Applying Mathematical Processes *project – see http://www.nuffieldcurriculumcentre.org/*

Follow-up task for students

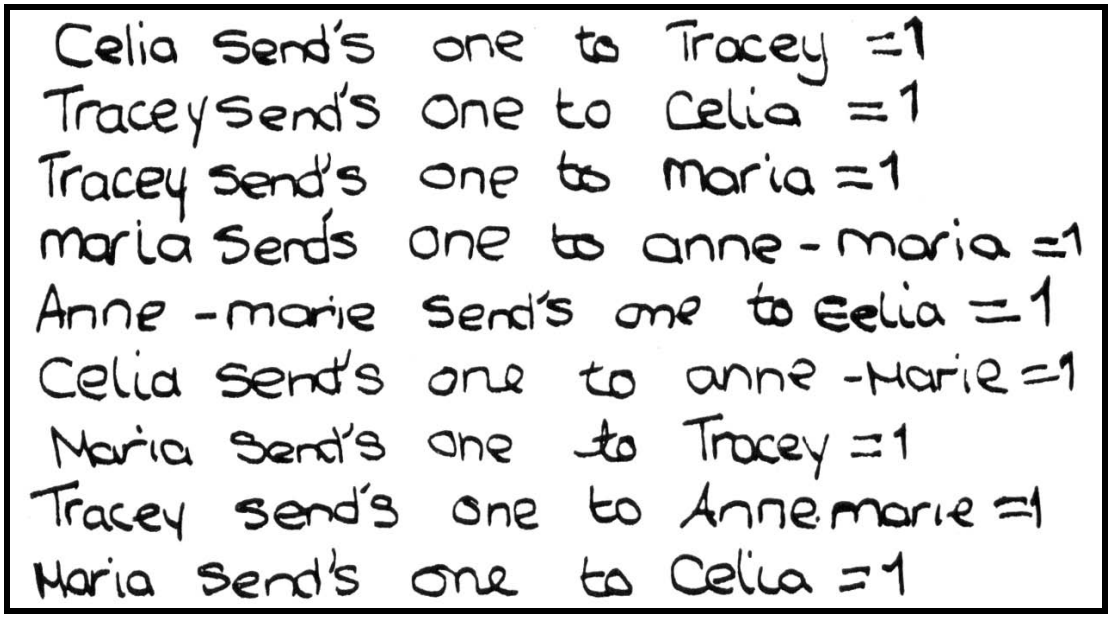
Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

- Have they chosen a sensible method?
- Are the calculations correct?
- Are the conclusions sensible?
- Is the work easy to understand?

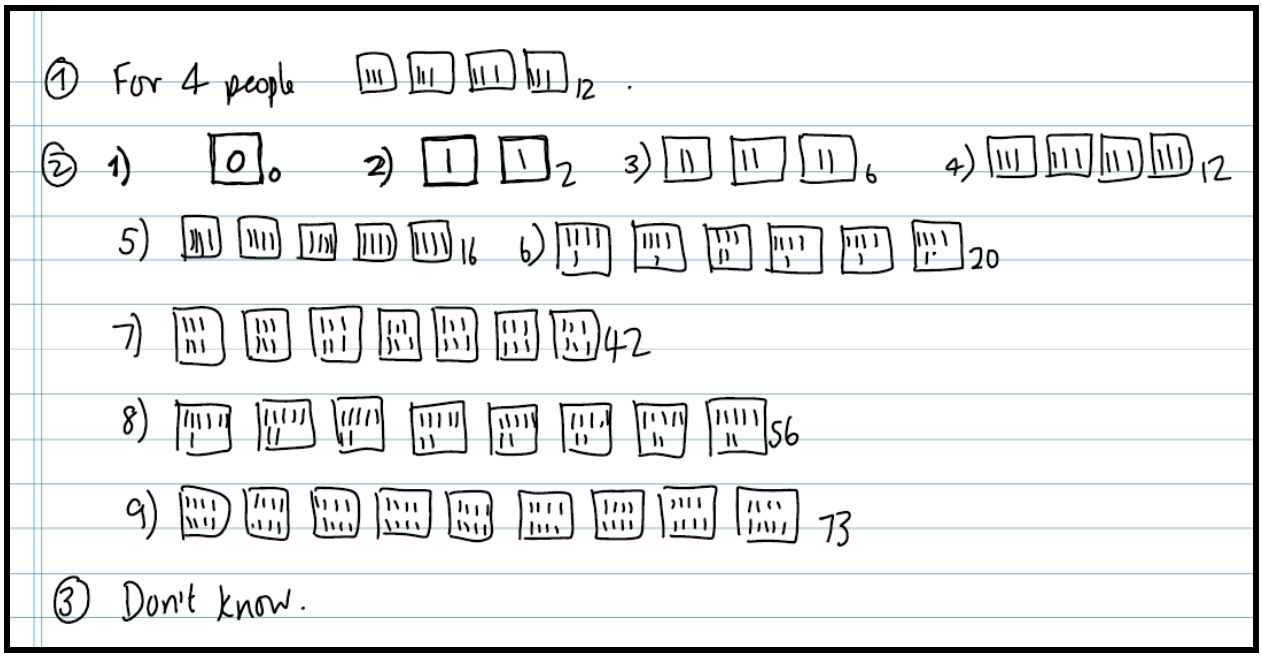
Name	Comments
Tom	
Sam	
Chris	
Lily	
Marvin	

Now try to write out an answer that is better than all of them!

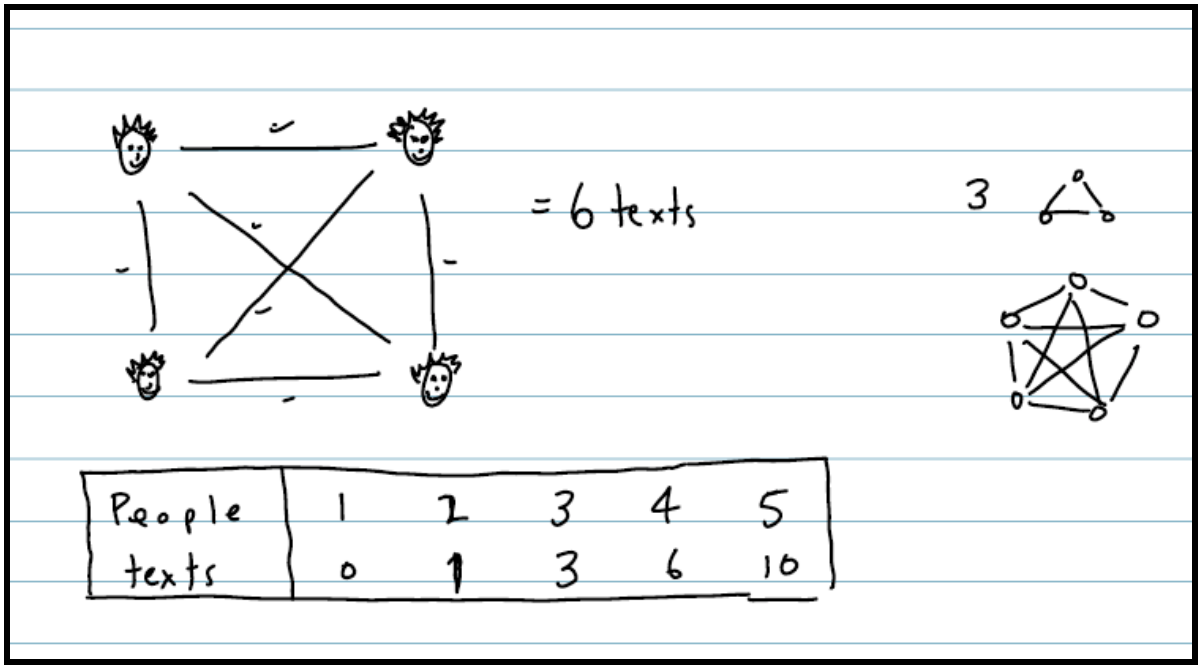
Tom's answer



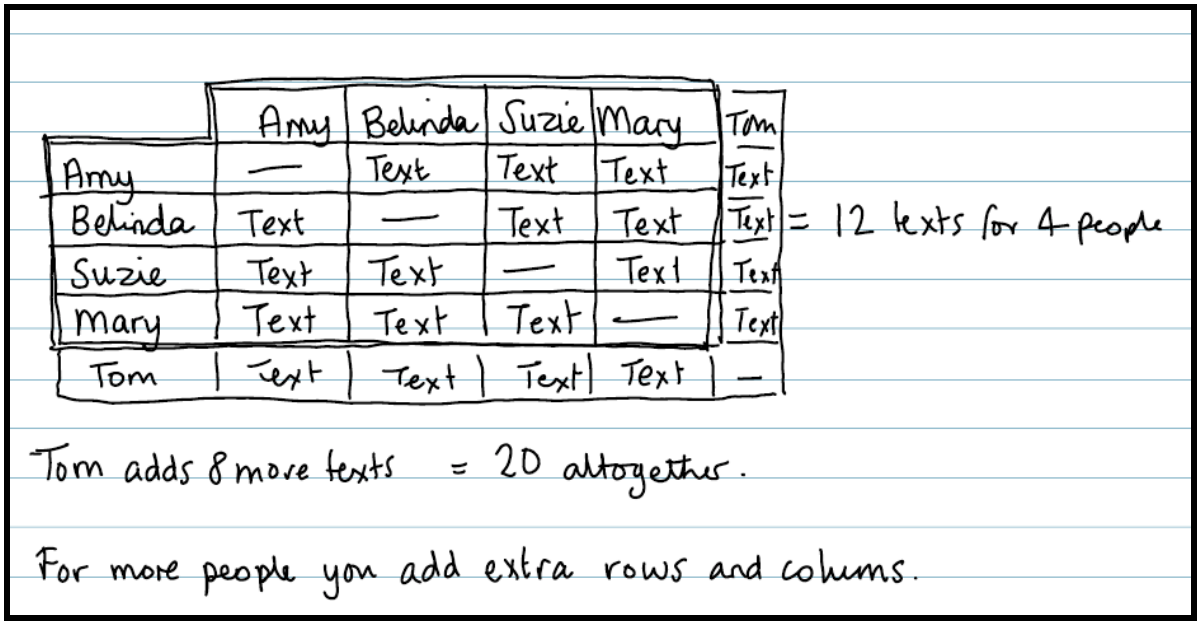
Sam's answer



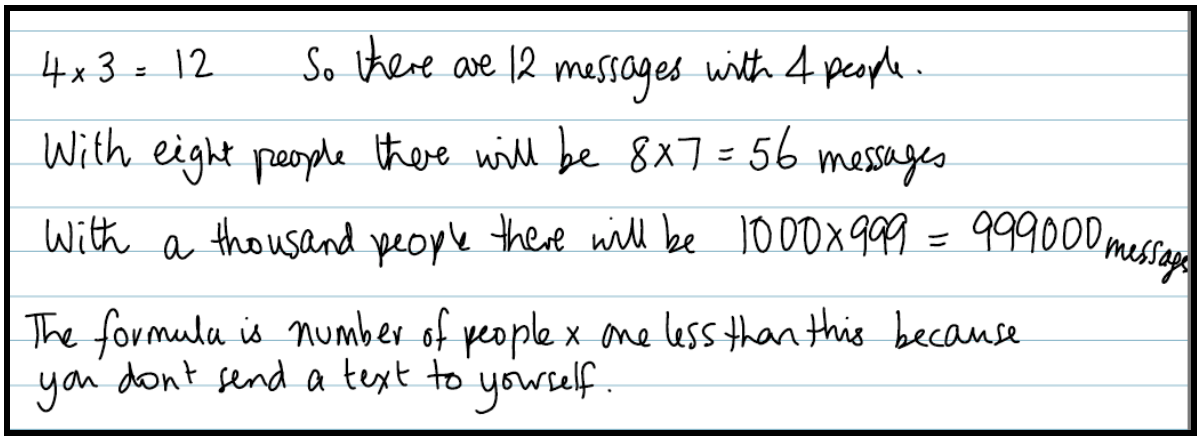
Chris's answer



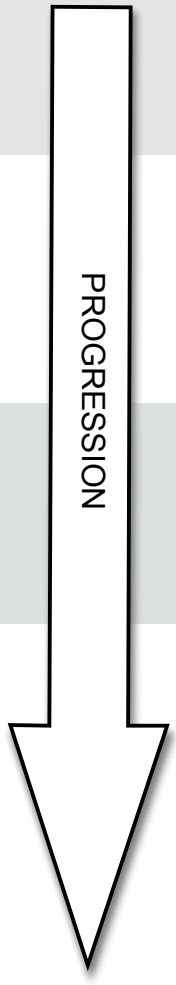
Lily's answer



Marvin's answer



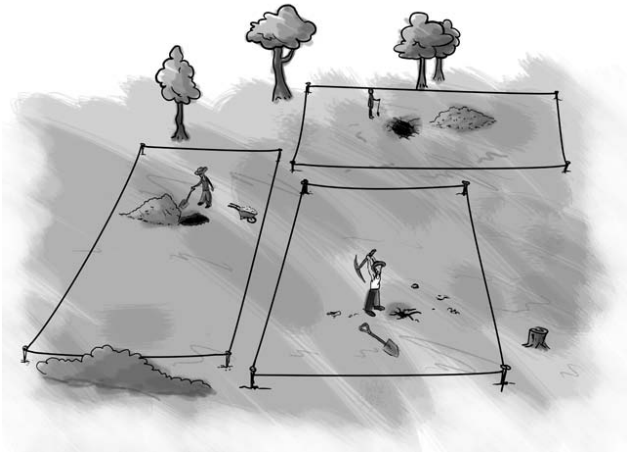
Progression in key processes



	Representing	Analysing	Interpreting and evaluating	Communicating
	Represents some individual text messages that are sent.	Works out the number of text messages for four people correctly.	Says that everyone sends the same number of messages.	Shows how the answer was found.
	Uses marks or diagrams to show the texts. Chooses to use repeated addition.	Increases the number of people in an organised way. Correctly works out the number of texts sent to different numbers of people.		Shows the method clearly and where the answers come from.
	Chooses to use multiplication to work out the number of texts sent.	Finds a correct pattern in the results. The reasoning is based on particular examples.	Explains the result for a number of people other than 4. Finds a correct rule for calculating the number of texts.	Explains how the rule links to the context of sending texts.
	Chooses to use algebra to show the general case.	The reasoning moves from looking at particular examples to more general cases.	Makes and justifies correct general statements relating the number of texts to the number of people. Makes and justifies statements for large numbers of students.	Writes a complete and concise summary with clear links to the original context. Discusses mathematical similarities and differences between sending texts and other contexts – e.g. matches in a football league.

Golden Rectangles

In the 19th century, many adventurers travelled to North America to search for gold.
A man named Dan Jackson owned some land where gold had been found.
Instead of digging for the gold himself, he rented plots of land to the adventurers.



Dan gave each adventurer four wooden stakes and a rope measuring exactly 100 metres.

Each adventurer had to use the stakes and the rope to mark off a rectangular plot of land.

1. Assuming each adventurer would like to have the biggest plot, how should he place his stakes?
   Explain your answer.

Read the following proposition:

“Tie the ropes together! You can get more land if you work together than if you work separately.”

2. Investigate whether the proposition is true for two adventurers working together, still using four stakes.

3. Is the proposition true for more than two people?
   Explain your answer.

Follow-up task for students

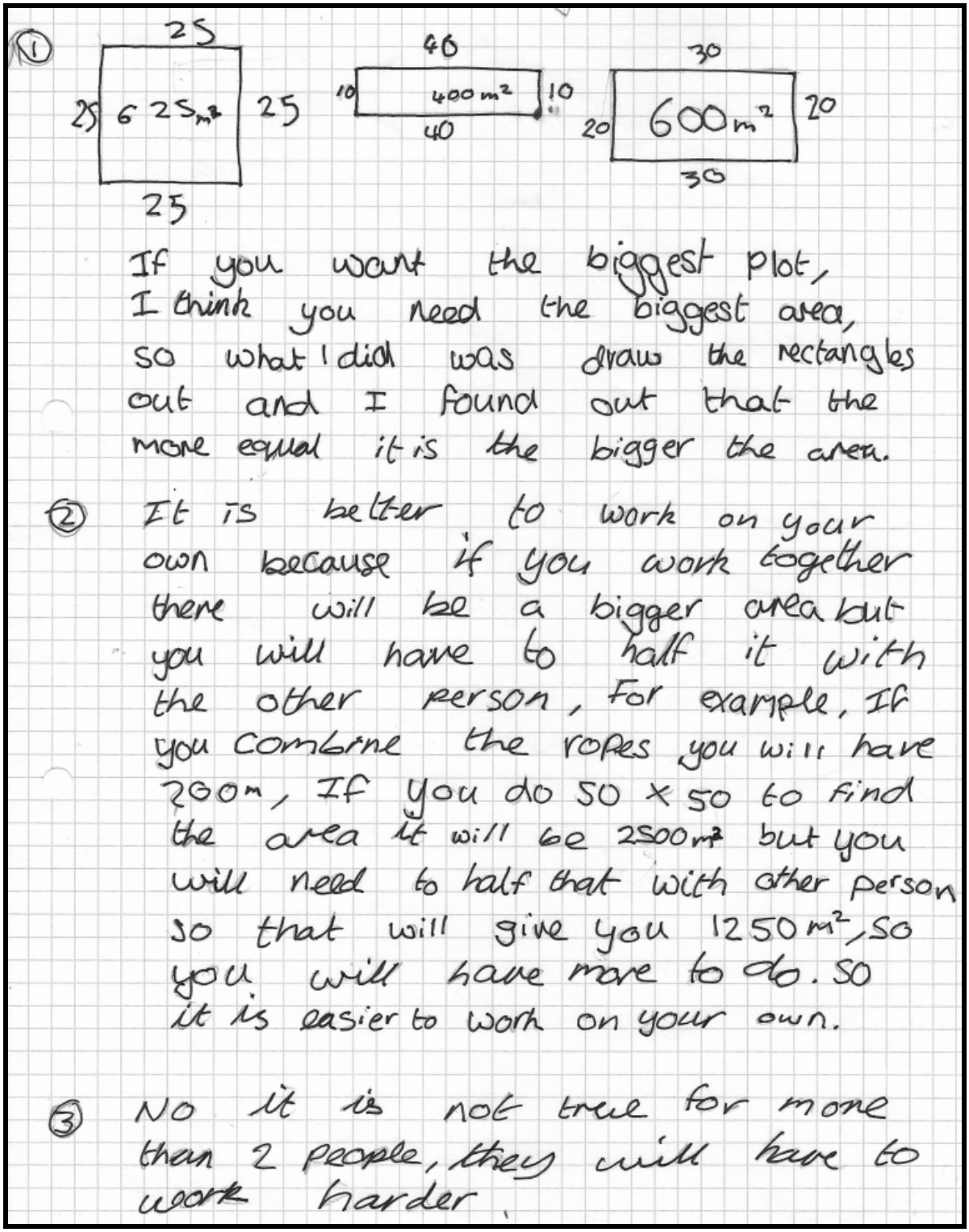
Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

- Have they chosen a sensible method?
- Are the calculations correct?
- Are the conclusions sensible?
- Is the work easy to understand?

Name	Comments
Alvin	
Bernie	
Chris	
Danny	
Elsie	

Now try to write out an answer that is better than all of them!

Alvin's answer

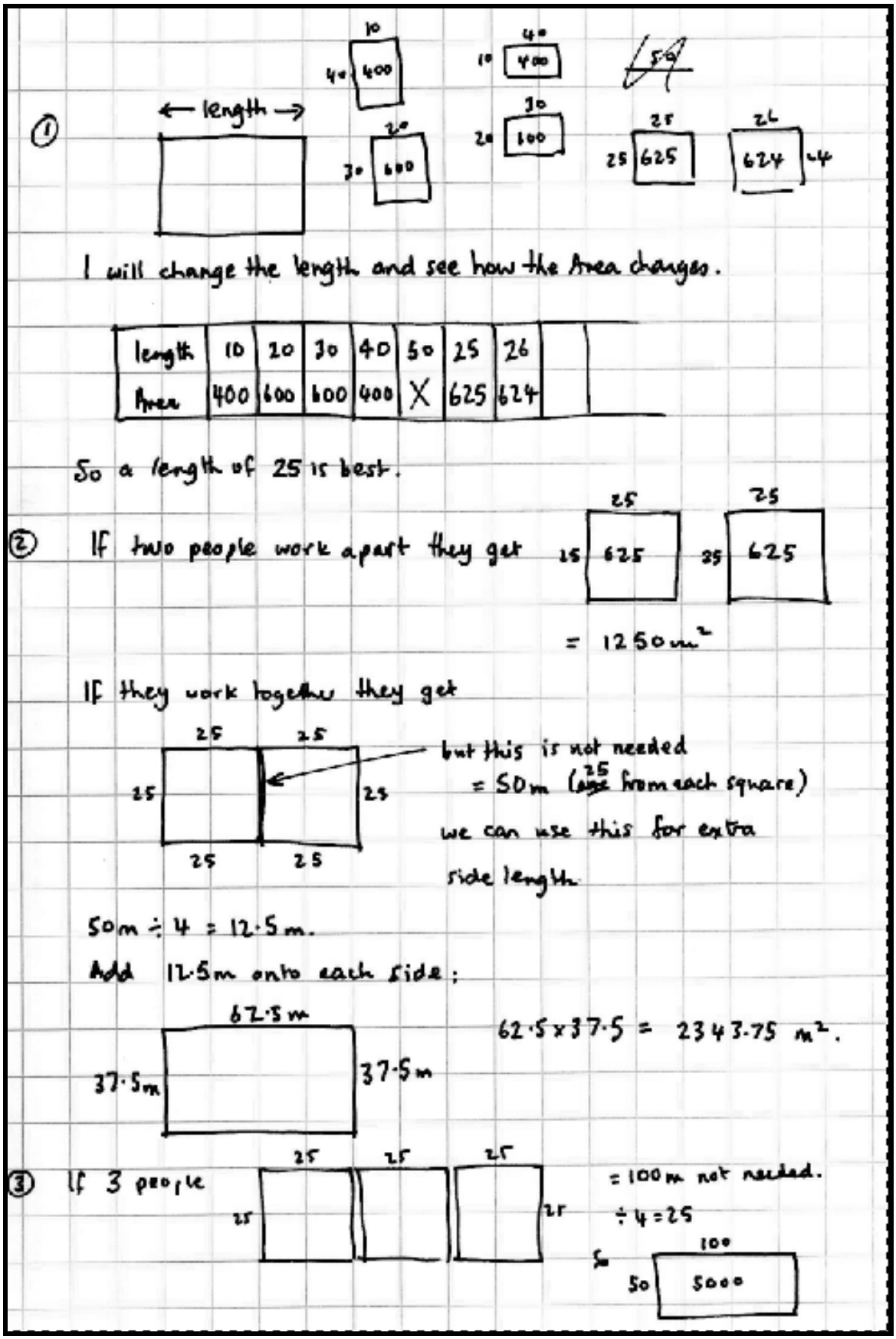


If you want the biggest plot, I think you need the biggest area, so what I did was draw the rectangles out and I found out that the more equal it is the bigger the area.

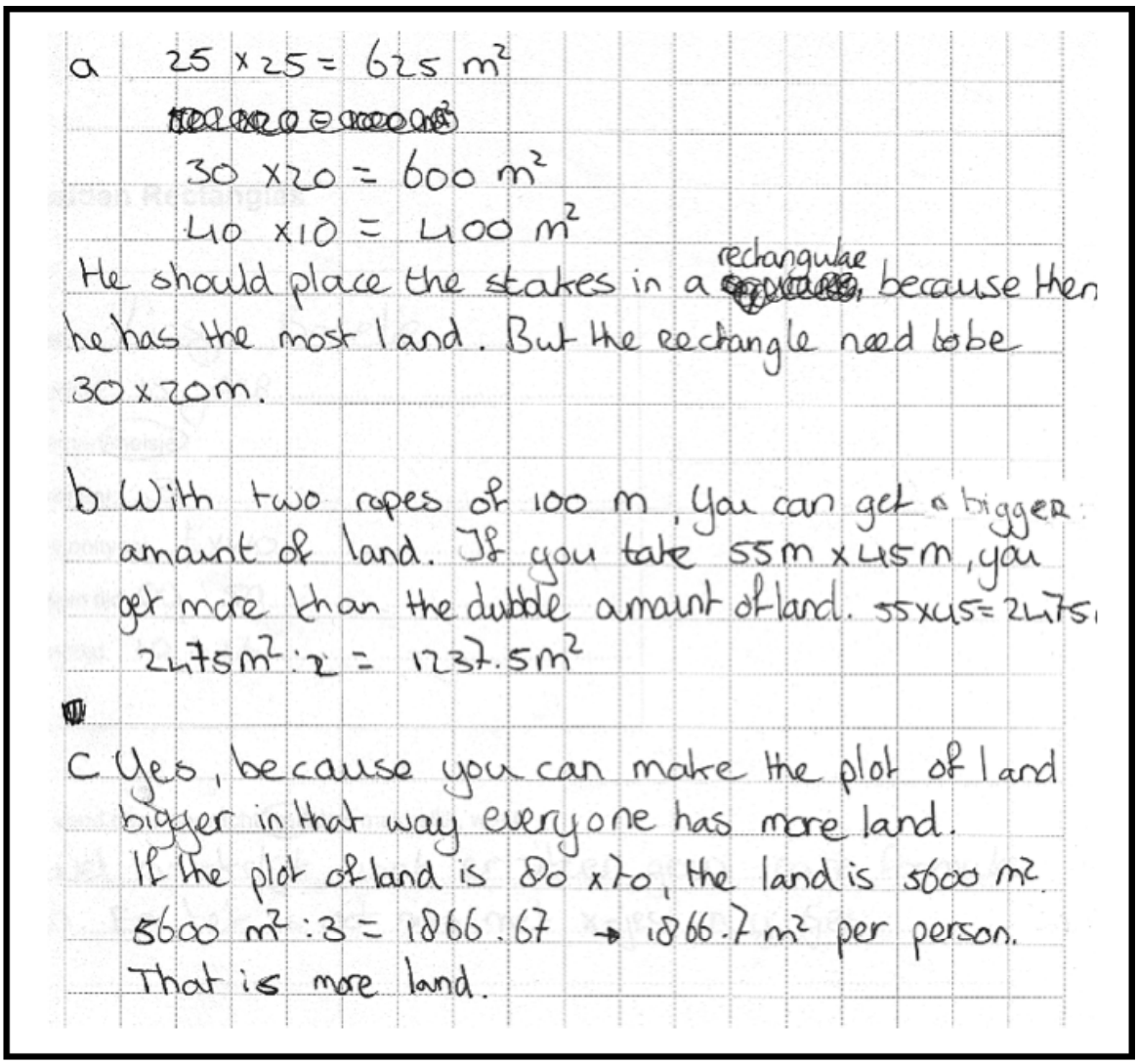
② It is better to work on your own because if you work together there will be a bigger area but you will have to half it with the other person, For example, If you combine the ropes you will have 200m, If you do 50 x 50 to find the area it will be 2500m² but you will need to half that with other person so that will give you 1250m², so you will have more to do. So it is easier to work on your own.

③ NO it is not true for more than 2 people, they will have to work harder.

Bernie's answer



Chris's answer



a $25 \times 25 = 625\ m^2$

$30 \times 20 = 600\ m^2$

$40 \times 10 = 400\ m^2$

He should place the stakes in a rechangulae because then he has the most land. But the rechangle need to be $30 \times 20m$.

b With two ropes of 100 m, you can get a bigger amount of land. If you take $55m \times 45m$, you get more than the dubble amount of land. $55 \times 45 = 2475$,

$2475\ m^2 : 2 = 1237.5\ m^2$

c Yes, because you can make the plot of land bigger in that way everyone has more land.

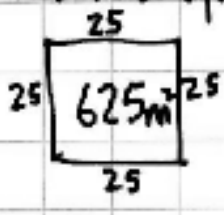
If the plot of land is 80×70, the land is $5600\ m^2$.

$5600\ m^2 : 3 = 1866.67 \rightarrow 1866.7\ m^2$ per person.

That is more land.

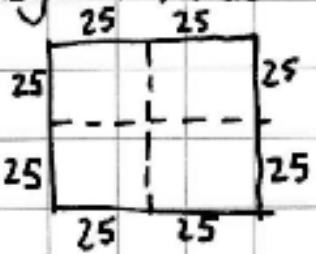
Danny's answer

① He should place his stakes in a square to give the biggest area like this



② If two adventurers work together they will have 200m² of rope so they can make a square twice as long and wide.

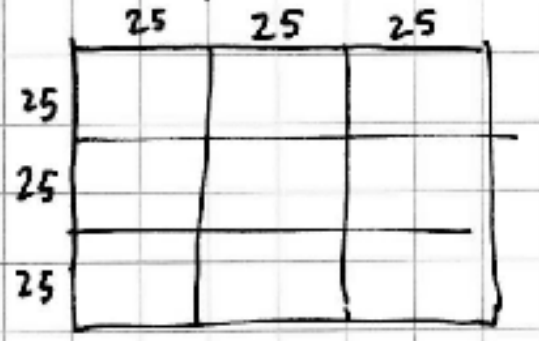
= 4x area.



This is much better than 2x area.

③ If three work together they will have 300m² of rope so they can make a square three times as long and wide

= 9x area



This is much better than 3x area.

I think that the area goes up by square numbers each time.

Elsie's answer

a: 4 x 25 metres → area = 25 x 25 = 625 m²
2 x 20 & 2 x 30 → area = 20 x 30 = 600 m²
2 x 10 & 2 x 40 → area = 10 x 40 = 400 m²

So 4 x 25 metres would make the biggest area

b 2 x 100 metres of rope = 200 m.
4 x 50 metres → area = 50 x 50 = 2500 m²
2 x 20 & 2 x 80 → area = 20 x 80 = 1600 m²
2 x 30 & 2 x 70 → area = 30 x 70 = 2100 m²
2 x 40 & 2 x 60 → area = 40 x 60 = 2400 m²
2 x 10 & 2 x 90 → area = 10 x 90 = 900 m²

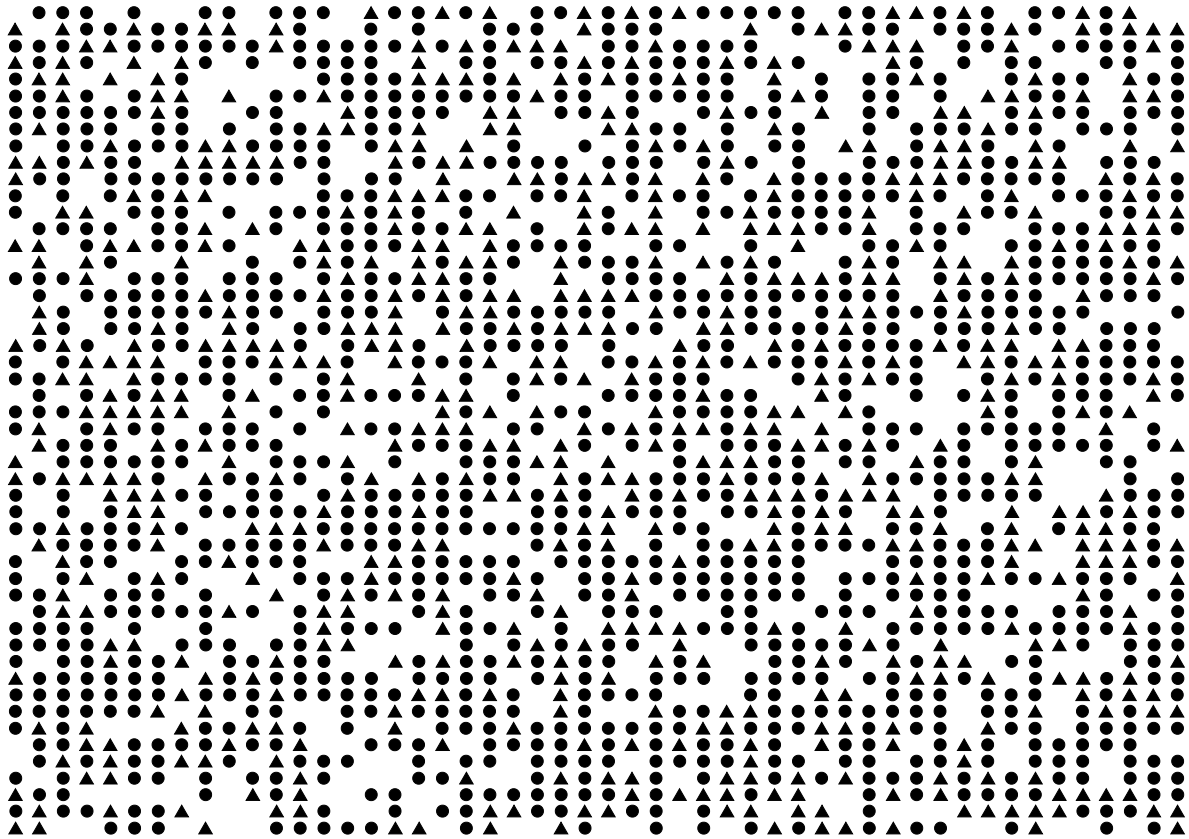
So the proposition is true, working together will deliver much more land to dig for gold.

c for example: 300 metres of rope
4 x 75 metres → area = 75 · 75 = 5625 m²
So how longer the rope is, how bigger the land will be.
400 metres of rope (4 people working together)
4 x 100 metres → area = 100 · 100 = 10000 m²

Progression in key processes

	Representing	Analysing	Interpreting and evaluating	Communicating
PROGRESSION	The student draws one or two rectangles with a perimeter of 100m.	The student works out the areas of their rectangles correctly.	The student draws several rectangles but not a square and the justification is incorrect or omitted.	The work is communicated adequately, but there are gaps and/or omissions.
	Draws several rectangles.	Calculates the areas of their rectangles and attempts to come to some generalisation.	Realises that different shapes have different areas but comes to incorrect or incomplete conclusion.	The work is communicated clearly and the reasoning may be followed.
	Draws several, correct rectangles for an adventurer working alone and for 2 working together. May draw far too many rectangles.	Calculates the areas correctly and finds that a square is best for 1 adventurer and that 2 working together do better than alone.	Attempts to give some explanation for their findings.	The work is communicated clearly and the reasoning may be easily followed.
	Draws an appropriate number of rectangles and collects the data in an organised way.	Calculates the correct areas, finds that a square is best for 1 adventurer and that 2 working together do better than alone. Finds a rule or pattern in their results.	Gives reasoned explanations for their findings.	Explains work clearly and may consider other shapes.

Counting Trees



This diagram shows some trees in a plantation.

The circles ● show old trees and the triangles ▲ show young trees.
Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

1. What method could he use to estimate the number of trees of each type? Explain your method fully.

2. On your worksheet, use your method to estimate the number of:
   (a) Old trees
   (b) Young trees

Follow-up task for students

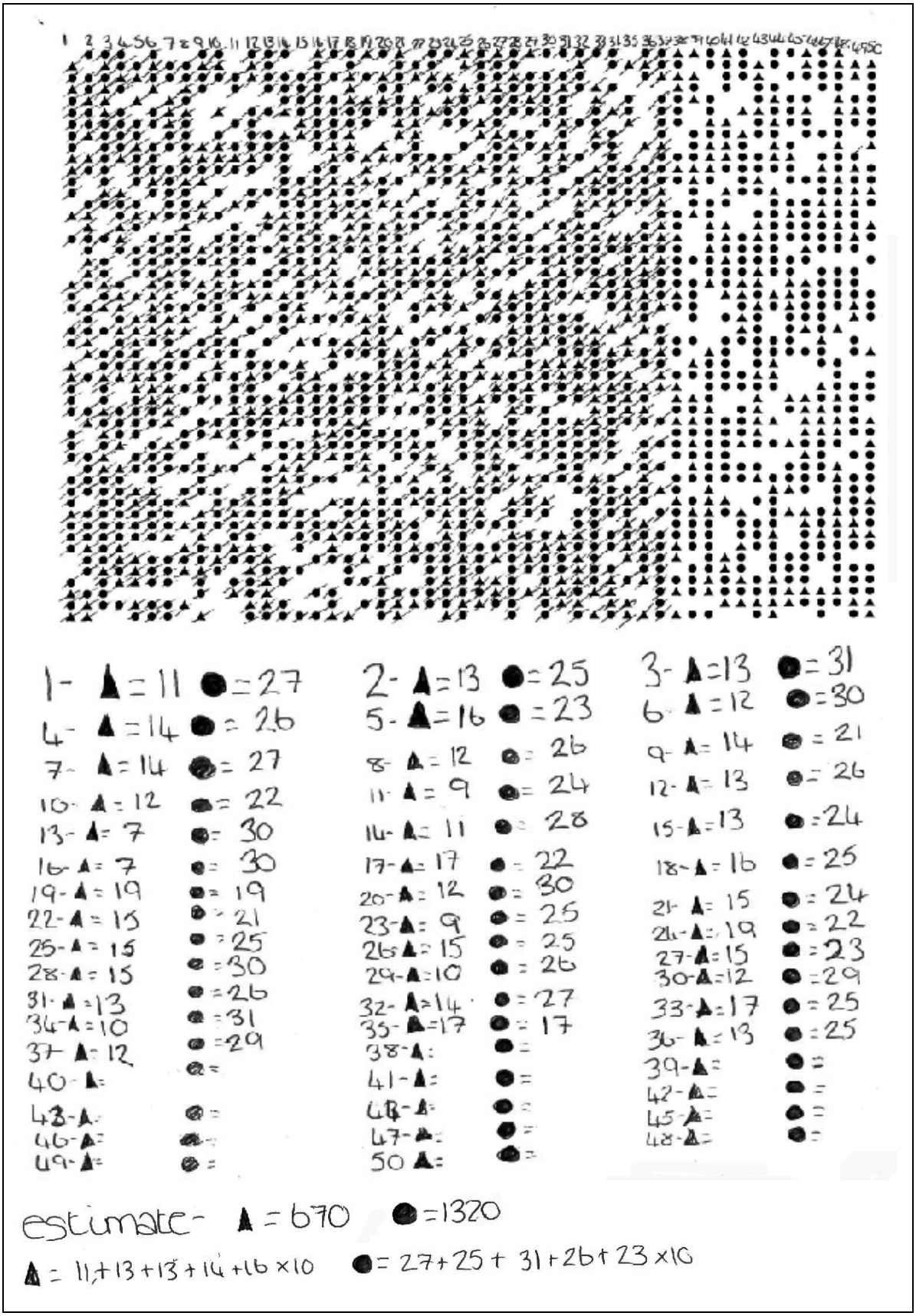
Look carefully at the following extracts of work from other students. Imagine you are their teacher. Go through each piece of work and write comments on each one.

- Have they chosen a sensible method?
- Are the calculations correct?
- Are the conclusions sensible?
- Is the work easy to understand?

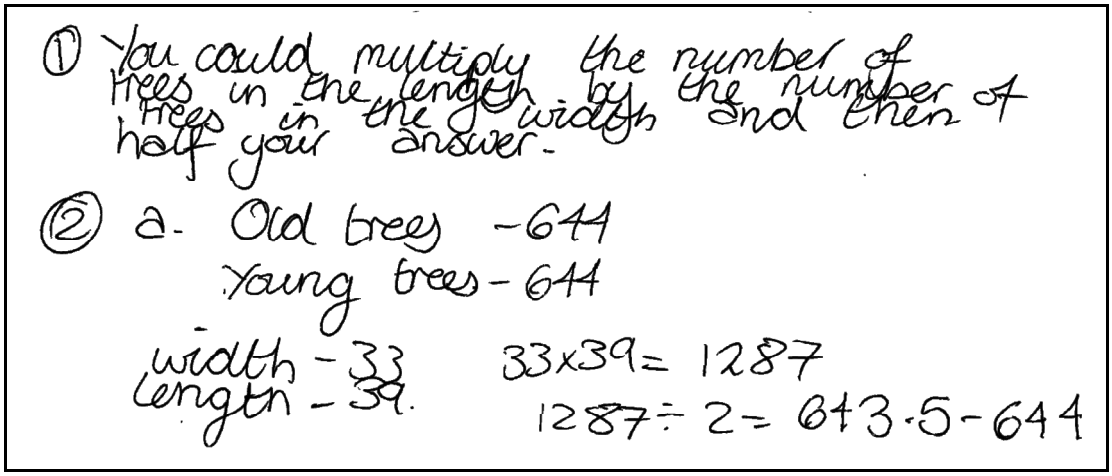
Name	Comments
Sarah	
Laura	
Jenny	
Woody	
Amber	

Now try to write out an answer that is better than all of them!

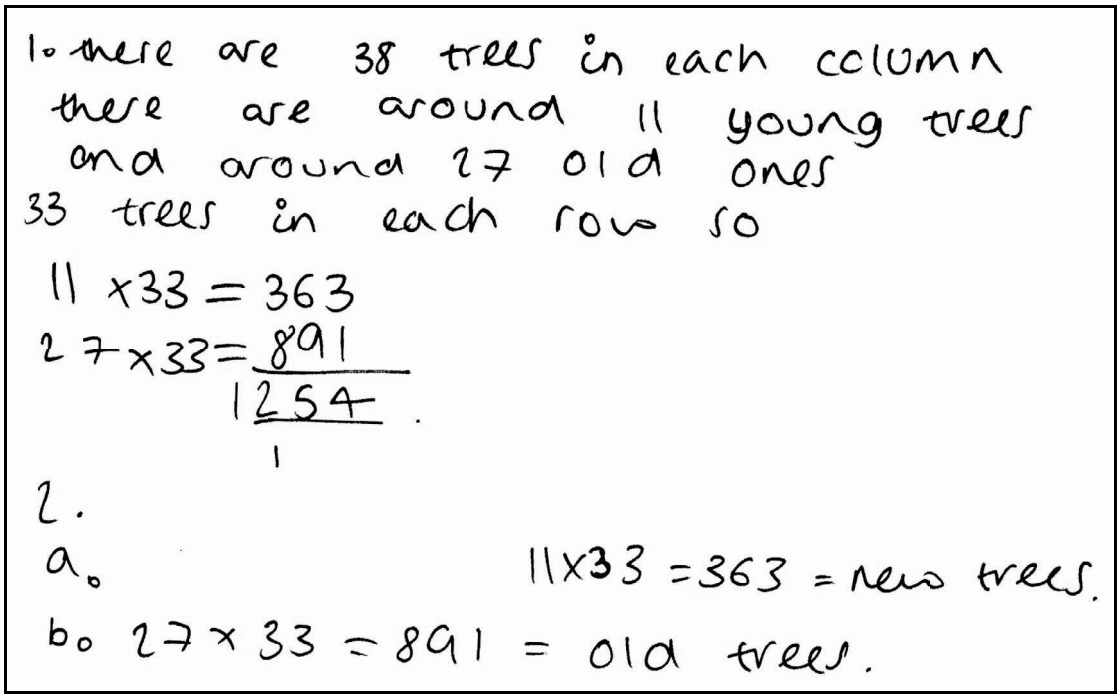
Sample response: Sarah



Sample response: Laura



Sample response: Jenny



Sample response: Woody

2 columns has 21 young trees
55 old

50 columns is approx
50 ÷ 2 = 25
25 × 21 = amount of young trees = 525
25 × 55 = amount of old trees = 1,375
rounded up

young 530
old 1,380

Sample response: Amber

Counting trees

1. If Tom draws a 10X10 square round some trees and counts how many old and new there are. There are 50 rows and 50 columns altogether so he must multiply by 25. He could do this a few times to check and then take the average.

2.

53 old	x 25	=	1325 old
28 new	x 25	=	700 new
19 spaces	x 25	=	475 spaces
100			2500

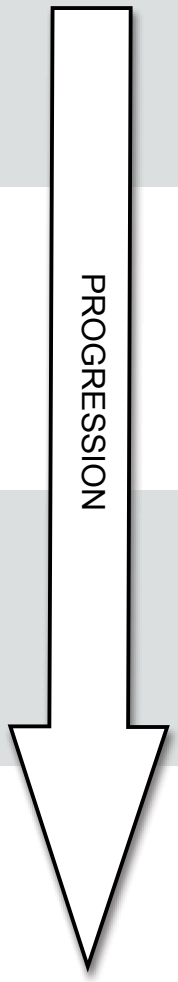
1325 + 1200 ÷ 2 = 1262.5
700 + 875 ÷ 2 = 787.5

Check

48 old	x 25	=	1200 old
35 new	x 25	=	875 new
17 spaces	x 25	=	425 spaces
100			2500

So about 1263 old trees and 788 new trees

Progression in key processes



	Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
	Chooses a method, but this may not involve sampling. E.g. Counts all trees or multiplies the number of trees in a row by the number in a column.	Follows chosen method, possibly making errors. E.g. Does not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due to method and errors.	Communicates work adequately but with omissions.
	Chooses a sampling method but this is unrepresentative or too small. E.g. tries to count the trees in first row and multiplies by the number of rows.	Follows chosen method, mostly accurately. E.g. May not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due mainly to the method.	Communicates reasoning and results adequately, but with omissions.
	Chooses a reasonable sampling method.	Follows chosen method, mostly accurately.	Estimates a reasonable number of old and new trees in the plantation. The reasonableness of the estimate is not checked. E.g. by repeating with a different sample.	Explains what they are doing but explanation may lack detail.
	Chooses an appropriate sampling technique.	Follows chosen method accurately. Uses a proportional argument correctly.	Deduces a reasonable number of old and new trees in the plantation. There is some evidence of checking the estimate. E.g. Considers a different sampling method.	Communicates reasoning clearly and fully.