BOWLAND
MATHS
Professional development

Assessing Key Processes to inform learning

How can I assess and build on what pupils are learning?

Module overview

Assessing pupils' progress as they develop the Key Processes is far from straightforward. This module considers the different ways this can be done and discusses the following questions:

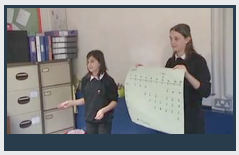
- How can Bowland assessment tasks be used to assess performance and progression in the Key Processes?
- How can this assessment be used to promote learning?
- What kinds of feedback are most helpful for pupils and which are unhelpful?
- How can assessment information be used to provide evidence for periodic assessment?

As questioning is a vital component to formative assessment, we recommend that you work on the CPD module *Questioning and reasoning*, before tackling this one.

Introductory session — 1 hour

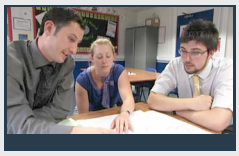
- Consider principles for assessment
- Analyse pupils' responses to an assessment task
- Observe formative assessment in action
- Plan an assessment lesson

Into the classroom — 1 hour



- Before the lesson, pupils attempt the task unaided.
- Prepare constructive feedback for pupils
- Pupils work alone, responding to the feedback
- Pupils work in pairs to improve their solutions
- Pupils share their approaches with the class
- Pupils continue with the problem or an extension of it

Follow-up session — 1 hour



- Report and reflect on the assessment lesson
- Consider the effects of feedback on pupil learning.
- Use "Progression Steps" to assess learning
- Discuss the use of periodic reviews in planning.
- Plan assessment strategies for future lessons

Resources needed

Handout 1	Some principles for assessing Key Processes
Handout 2	Three assessment tasks and four sample responses on each
Handout 3	Improving pupils' responses through questioning
Handout 4	Questioning that will help improve responses
Handout 5	The effects of feedback on pupils' learning
Handout 6	Progression steps in the Key Processes
Handout 7	Suggestions for further reading

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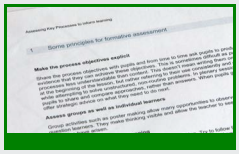
Introductory session

Introduction — How can we assess pupils' use of the Key Processes? — 5 minutes

Pupils enjoy the challenge of tackling tasks that develop Key Processes, but attempting to assess and describe their progress is difficult. Processes are less well defined than content, and pupils' success in applying them will vary considerably from problem to problem.

In this module, three teachers, Amy, Andrew and Dominic, explore how the Bowland assessment tasks may be used to help assess pupils' progress and provide feedback that promotes learning.

Activity 1 — Consider principles for assessment — 10 minutes



Handout 1 presents some principles that have been found helpful in assessing processes.

- Which of these principles do you most frequently use?
- Which of these principles do you neglect? Why is this?

Activity 2 — Analyse pupils' responses to an assessment task — 20 minutes

Handout 2 presents three Bowland Assessment tasks together with four pupil responses on each. The tasks are: *Counting Trees, Cats and Kittens, Security Cameras.*

Read through all three tasks then choose one task that will be most suitable for a class you will soon teach. If you are working on this module in a group, it will be helpful if each participant chooses the same problem, as this will facilitate the follow-up discussion.

Consider the four pupil responses.

- What does each pupil's response tell you about his or her capacity to use each of the Key Processes: *represent, analyse, interpret and evaluate, communicate and reflect*?

Handout 3 offers some comments on pupils' responses to each of the tasks.

- If you were the teacher of these pupils, what feedback would you give them, to help them improve their responses? Try to frame this help in the form of oral questions you could ask in the classroom. You may find it helpful to refer to the generic questions given on Handout 4 .

Activity 3 — Observe formative assessment in action — 15 minutes

In this activity, you are provided with video extracts of Amy, Andrew and Dominic exploring how formative assessment may be used to promote pupils' learning. They are using the three tasks from Activity 2.

In an earlier lesson, these teachers had asked pupils to sit in different places and attempt one of the tasks individually, with no help. They then collected in their pupils' responses, assessed the work qualitatively and prepared written feedback in the form of questions. The film clips you are about to see are taken from the follow-up lesson. Pupils have returned to their normal places and most have solutions that are different to those of their partners.

Watch the video and consider:

- What different kinds of assessment can you see?
- What is the purpose of each kind of assessment?
- What do both the teachers and pupils learn?

In the video, you will see:

- Andrew exploring how pupils respond to his feedback on the "counting trees" problem;
- Amy listening to, then questioning individuals as they try to share their ideas and produce joint solutions to the "security camera" problem;
- Dominic listening to presentations from pupils on their methods and reasoning for the "cats and kittens" problem;
- Amy concluding her lesson by asking pupils to describe how they have used her feedback to improve their work.

Activity 4 — Plan an assessment lesson — 10 minutes

You may now like to plan your own lesson using one of the problems.

- Plan a time for pupils to tackle the problem on their own without help.
- Plan how you will assess this work, give feedback and conduct a follow up lesson.
- Collect samples of pupils' work to show how their thinking has changed. These will be discussed at the follow-up session.

To help you plan your lesson, you may find it helpful to watch the video clips and refer to the lesson plan provided.

This is the end of the *Introductory session*. After you have tried out your lesson with your own pupils, return for the *Follow-up session*. Resources to support the lessons, and suggested lesson plans, can be found in the *Into the classroom* session.

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How can I assess and build on what pupils are learning?

Into the classroom

The following suggestions describe one possible approach to self- and peer-assessment. Pupils are given a chance to tackle a problem unaided, to begin with. This gives you a chance to assess their thinking and to identify pupils that need help. This is followed by a formative lesson in which they collaborate, reflect on their work and try to improve it.

Pupil handouts and presentations for these tasks can be found in the *Into the classroom* section on the DVD or Website.

Before the lesson — 20 minutes

Before the lesson, perhaps at the end of a previous lesson, ask pupils to attempt one of the assessment tasks, *Counting Trees, Cats and Kittens or Security Cameras* on their own. Pupils may need calculators, pencils, rulers, and squared paper.

The aim is to see how able you are to tackle a problem without my help.

- *You will not be told which bits of maths to use.*
- *There are many ways to tackle the problem - you choose.*
- *There may be more than one 'right answer'.*

Don't worry if you cannot understand or do everything because I am planning to teach a lesson on this next in the next few days.

Make sure that pupils are familiar with the context of the problem.

Counting Trees
Does anyone know what a tree plantation is?
How is a plantation different from a natural forest?
The plantation consists of old and new trees
How might the arrangement of trees in a plantation differ from that of a natural forest?

Cats and Kittens
This is a poster made by a cats' charity, encouraging people to have their cats spayed so they can't have kittens. The activity is about what happens if you don't have your cat spayed and whether the statement on the poster is correct.
Is it realistic that one female cat would produce 2000 descendants in 18 months?
You are given some facts about cats and kittens that will help you decide.

Security Cameras
Have you ever seen a security camera in a shop or a bus? What did it look like?
Some may not look like cameras at all, but rather like small hemispheres. They may be fixed, but many swivel round. The cameras in this problem can turn right round through 360°. The drawing shows a plan view of a shop.
This means we are looking down on the shop from above.
The little circles represent people standing in the shop.

Remember to show your working so I can understand what you are doing and why.

Collect in their work and provide constructive, qualitative feedback on it. This should focus on getting pupils to think and reason - a Key Process agenda. Don't give marks or levels! Write *only* questions below their work. Focus feedback on such issues as:

Representing
Can you think of a different way of tackling his problem?
What sort of diagram might be helpful?
What assumptions have you made?

Reasoning
How have you got this result?
Have you checked your calculations?
What would happen if ...?

Interpreting
How can you test the accuracy of your estimate?
What other sample could you have chosen?

Communicating
I find it difficult to follow your thinking here.
Can you present your reasoning so that someone else can follow every step?

Try to identify particular pupils who have struggled and who may need support. Also look out for pupils that have been successful. These may need an extension activity to further challenge them.

Resources needed for the lesson

You will need the following resources:

- One copy of the problem sheet per pupil
- Mini whiteboards
- Large sheets of paper for making posters and felt-tipped pens
- Calculators and rulers

Counting Trees

- Spare, large copies of the trees picture for groups to work on together.

Cats and Kittens

- A supply of graph paper or squared paper (if requested)

Security Camera.

- Spare copies of the plan of the shop for rough working
- Squared paper (only if requested)

Re-introduce the problem to the class 5 minutes

Begin the lesson by briefly reintroducing the problem:

> *Do you remember the problem I asked you to have a go at last time?*
> *I have had a look at your work and I have written some comments at the bottom of it.*
> *Today we are going to work together trying to improve on these initial attempts.*
> *First, on your own, carefully read through the questions I have written on your work.*
> *Use your mini-whiteboards to note down answers to these questions.*

It is helpful to ask pupils to write their ideas on a large sheet of paper or mini whiteboard using felt-tipped pen. This helps you monitor their work and also helps pupils to share their ideas later in the lesson.

Pupils work alone responding to your feedback 5 minutes

Allow the pupils some time to reflect on your comments and write their responses.

Pupils work in pairs to improve their solutions 10 minutes

Ask pupils to now work in pairs or threes. Give out a large sheet of A3 (at least) paper and a felt-tipped pen to each group.

> *Now I want you to share your work with a partner.*
> *Take it in turns to explain how you did the task and how you now think it could be improved.*
>
> *Now I want each pair to work together, comparing their ideas and the feedback I have given. Together, I want you to try to produce an answer to the problem that is better than each of you did separately.*

Go round the room, listening, assessing their thinking and making interventions asking strategic questions. Consult a copy of the progression steps for the relevant problem and decide which questions would be most appropriate for moving their thinking towards higher levels of performance. Use strategic questions like:

> *What is known and what is unknown?*
> *What are you asked to find out?*
> *How can we simplify this problem?*
> *What assumptions have you made?*

Pupils share their approaches with the class 15 minutes

Ask pupils to present their ideas and approaches to the class. Focus on their methods rather than their answers. As they respond, use the progression steps to assess their responses. In particular, focus on the quality of the reasoning and communication.

> *"We decided to count the different types of trees along each side, then multiply these numbers together."*
> *"We drew a time line along the top of the paper and then drew cats underneath to show when they gave birth."*

As pupils present their ideas, ask other pupils to comment on:

- Representing: Did they choose a good method?
- Analysing: Is the reasoning correct – are the calculations accurate?
- Interpreting: Are the conclusions sensible?
- Communication: Was the reasoning easy to understand and follow?

Pupils continue with the problem or an extension of the problem 20 minutes

Encourage pupils to return to the problem and continue working on it using some of the ideas that have been shared. If they have already produced a good solution, either ask them to find an alternative method, a more convincing reason, or to explore an extension.

> ***Counting Trees***
> *If I now showed you a very large jar of coloured sweets, how could you estimate the fraction that are red? Write down you method. Can you use what you learned from "Counting Trees"?*

> ***Cats and Kittens***
> *Can you find a simpler, more elegant way of presenting your calculations to "Cats and Kittens"? Can you use a diagram of some kind?*

> ***Security Camera***
> *There are several places that the camera might be placed that are as good as the one you have found. Try to find all the solutions. Can you convince me that these are all possible solutions? Can you explain why they all give the same coverage of the shop?*

Collect examples of pupils' work for the follow-up discussion. Try to assess how much pupils have learned from the sharing session.

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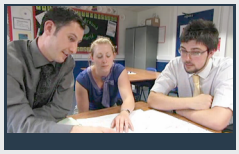
Follow-up session

Activity 1 — Report and reflect on the assessment lesson — 10 minutes



Take it in turns to share stories of the assessment strategies you used in your lessons. The sample work you have brought along will be discussed in Activity 3, below.

- How did you collect and assess evidence of pupils' use of the Key Processes?
- What did you learn from this evidence?
- What did pupils learn from the follow-up lesson?
- What are the implications for you mathematics teaching more generally?

Activity 2 — Consider the effects of feedback on pupils' learning — 15 minutes

So far we have focused on the teachers' role in providing assessment feedback to pupils. In this activity we will consider the use pupils make of different types of feedback and the impact this has on their learning.

Watch the video of Andrew's pupils as they discuss the impact of assessment feedback on their learning.

- Which of their comments strike you as particularly perceptive and important?
- What are the implications of their comments?

Compare their comments with the research quotes given on Handout 5.

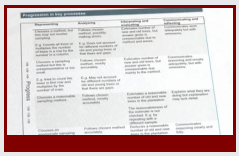
- What are the implications of these findings for your own practice?
- What would happen if you stopped giving marks or levels on pupils' work? Why are so many teachers resistant to making this change?
- What are the implications of giving qualitative feedback that *"concentrates on specific problems with their work, and gives them both a clear understanding of what is wrong and achievable targets for putting it right"?*
- Does this kind of feedback necessarily take much longer to give?

Handout 5 presents some results of research from Black and Wiliam (1998) into the relative merits of feeding back assessment information to pupils in different forms. In particular it compares the effects of feeding back quantitative information in the form of marks, levels and rankings with the effects of offering qualitative information in the form of specific, content-focused feedback.

Research shows that learners benefit most from feedback that:

- Focuses on the task, not on levels or marks.
- Is detailed rather than general.
- Explains *why* something is right or wrong.
- Is related to objectives
- Makes clear what has been achieved and what has not
- Suggests what the learner may do next
- Offers specific strategies for improvement

Activity 3 — Using the 'Progression Steps' to assess learning — 15 minutes



Look at Handout 6. For each task, we have provided progression steps that provide a framework for assessing pupils' use of the Key Processes. When solving a problem, the four processes are interrelated and need not be considered separately, but the framework approach is useful in helping us see how each process is embodied in the task.

- Try using the progression steps to assess your own pupils' work.
- How else might you use this framework to develop your pupils' understanding of the Key Processes?

If you have been unable to collect this work, then you may like to use the sample work provided in Handout 2 for this activity.

Teachers have found these progression steps useful when constructing feedback questions and comments on pupils' work. They have also been simplified and used directly with pupils to assist them in peer assessment. This will be discussed in the later module: *Involving pupils in self and peer assessment.*

Activity 4 — Discuss the use of periodic reviews in planning — 15 minutes

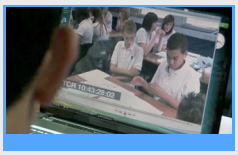
Day-to-day assessment provides a wide range of evidence of learning, in specific contexts, *which shapes immediate next steps*.

Periodic review of this evidence gives a clear profile of pupils' achievement across a whole subject and *informs and shapes future planning and targets for improvement*. (DCSF, 2008, p. 6)

- Consider how you might incorporate the Bowland assessment tasks into your normal scheme of work.
- How will you periodically collect evidence of your pupils' progress?
- How will you use this evidence to inform future planning and target setting?

On the video, Amy describes how one might, over time, collect a portfolio of each pupil's work on the Bowland tasks, and relate each to the progression steps. In this way both pupils and teachers can monitor the qualitative development of pupils' ability to *represent, analyse, interpret and communicate*. One might use this information to help select further problems that focus on the learning needs that become apparent. For example, if the pupil shows a consistent difficulty in communicating their reasoning, then a problem may be selected where this process is central.

Activity 5 — Plan assessment strategies for future lessons — 5 minutes



Conclude this module by discussing some ways of applying what you have learned in this PD module to the other mathematics lessons that you teach.

- How could you involve pupils in improving your assessment practices?

Assessment should go beyond the teacher giving guidance and feedback. It should be two-way. The final video clip shows the end of Amy's lesson in which she asks pupils to tell her the kinds of feedback that they have found most helpful.

Further Reading

See Handout 7 for suggested further reading.

References:

Black, P., & Wiliam, D. (1998). *Inside the black box : raising standards through classroom assessment*. London: King's College London School of Education 1998.

DCSF (2008). The Assessment for Learning Strategy. *DCSF-00341-2008*. Retrieved from http://www.teachernet.gov.uk/publications

1 Some principles for assessing Key Processes

Make the process objectives explicit

Share the process objectives with pupils and from time to time ask pupils to produce evidence that they can achieve these objectives. This is sometimes difficult as pupils find processes less understandable than content. This doesn't mean writing them on the board at the beginning of the lesson, but rather referring to their use consistently and explicitly while attempting to solve unstructured, non-routine problems. In plenary sessions, ask pupils to share and compare approaches, rather than answers. When pupils get stuck, offer strategic advice on what they need to do next.

Assess groups as well as individual learners

Group activities such as poster making allow many opportunities to observe, listen, and question learners. They make thinking visible and allow the teacher to see quickly where difficulties have arisen.

Watch and listen before intervening

Before intervening in a group discussion, wait and listen. Try to follow the line of reasoning that learners are taking. When you do intervene, begin by asking them to explain something. If they are unsuccessful then ask another learner to help.

Use divergent assessment methods ("Show me what you know about ...").

Convergent assessment strategies are characterised by tick lists and can-do statements. The teacher asks closed questions in order to ascertain whether or not the learner knows, understands or can do a predetermined thing. This is the type of assessment most used in written tests.

Divergent assessment, in contrast, involves asking open questions that allow learners opportunities to describe and explain their thinking and reasoning. These questions allow learners to surprise us - the outcome is not predetermined.

Give constructive, useful feedback

Research shows that responding to pupils' work with marks or levels is ineffective and may even obstruct learning. Quantitative feedback of this type results in pupils comparing marks or levels and detracts from the mathematics itself.

Instead, use qualitative oral and written comments that help learners recognise what they can do, what they need to be able to do and how they might narrow the gap.

Change teaching to take account of assessment

As well as providing feedback to learners, good assessment feeds forward into teaching. Be flexible and prepared to change your teaching plans in mid-course as a result of what you discover.

The above principles reflect the characteristics of *formative assessment,* which may be defined as:

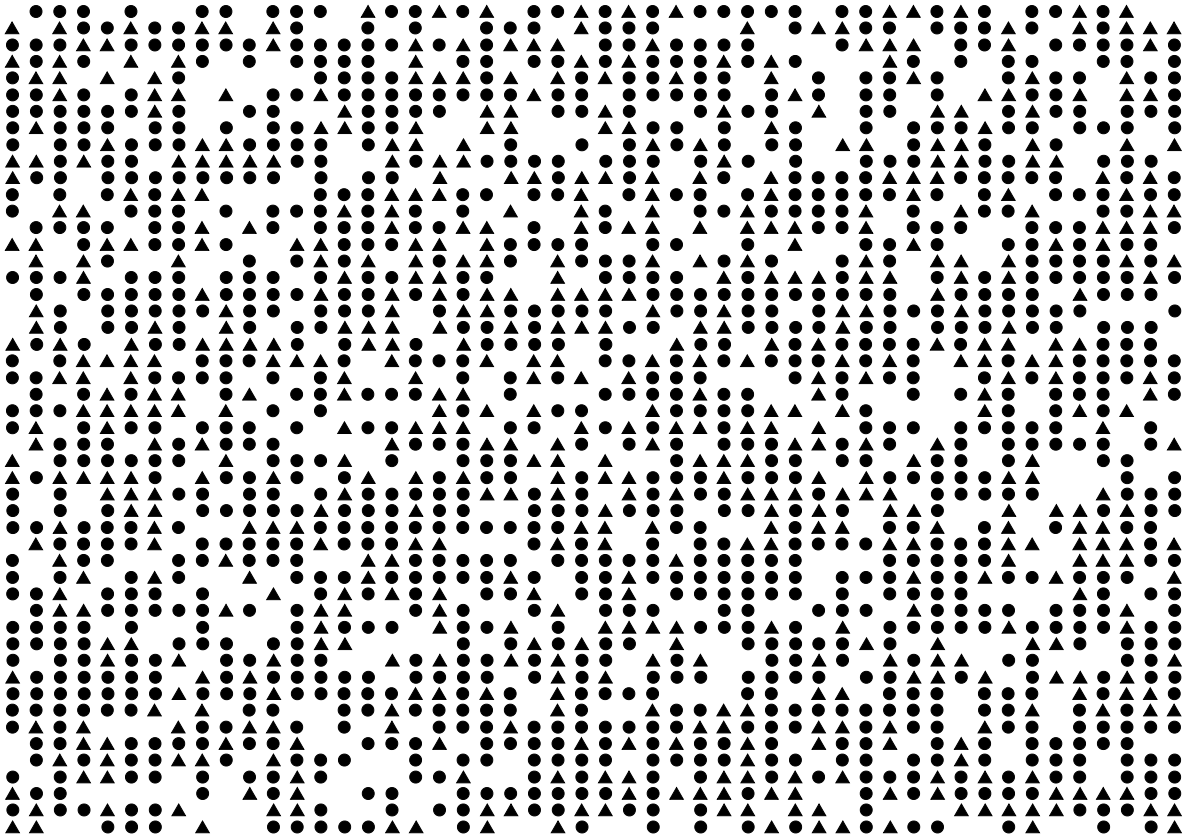
"… all those activities undertaken by teachers, and by their students in assessing themselves, which provide information to be used as feedback to modify the teaching and learning activities in which they are engaged. Such assessment becomes 'formative assessment' when the evidence is actually used to adapt the teaching work to meet the needs."

(Black & Wiliam, 1998 para, 91)

Adapted from: Improving Learning in Mathematics, Department for Education and Skills, 2005.

2 Three assessment tasks and four sample responses on each

Counting Trees



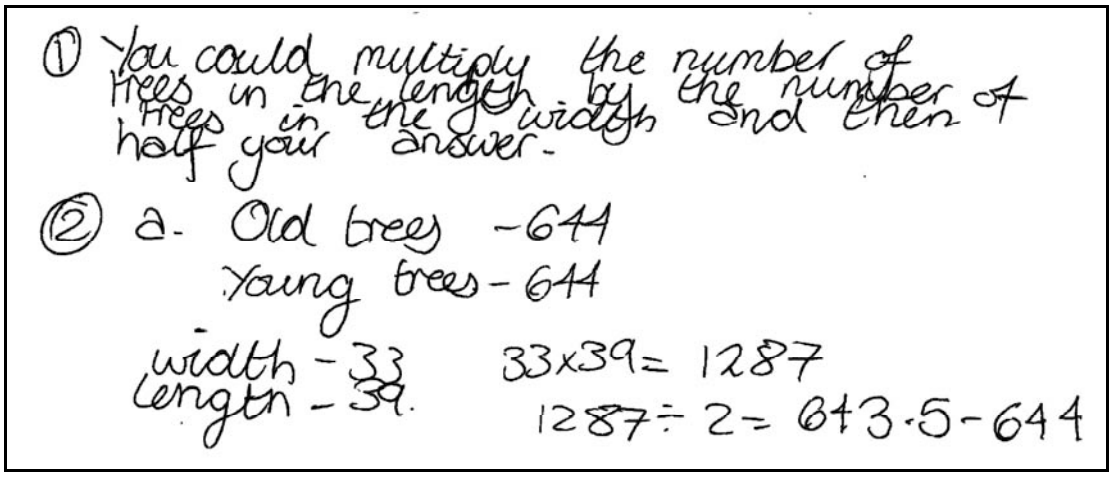
This diagram shows some trees in a plantation.

The circles ● show old trees and the triangles ▲ show young trees.
Tom wants to know how many trees there are of each type, but says it would take too long counting them all, one-by-one.

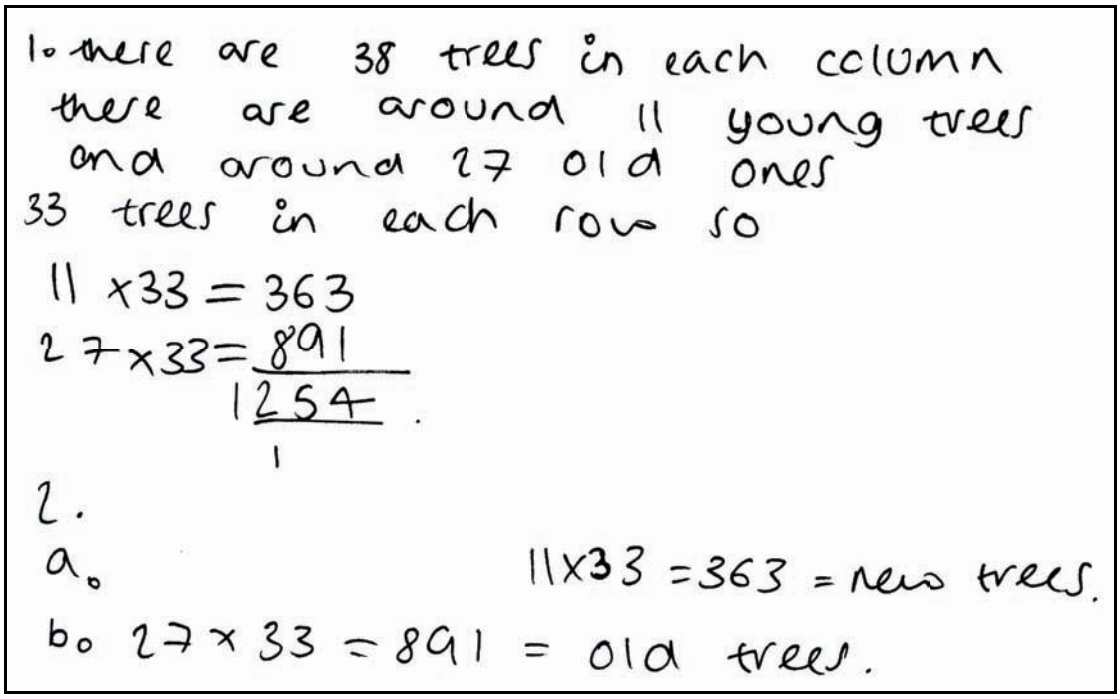
1. What method could he use to estimate the number of trees of each type? Explain your method fully.

2. On your worksheet, use your method to estimate the number of:
   (a) Old trees
   (b) Young trees

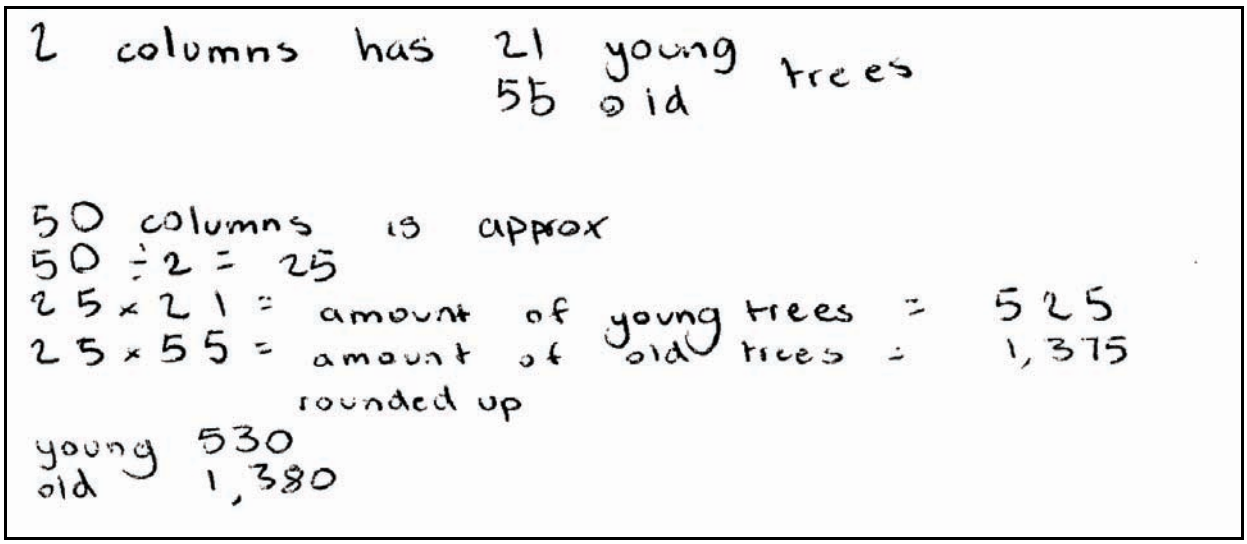
Sample response: Laura



Sample response: Jenny



Sample response: Woody

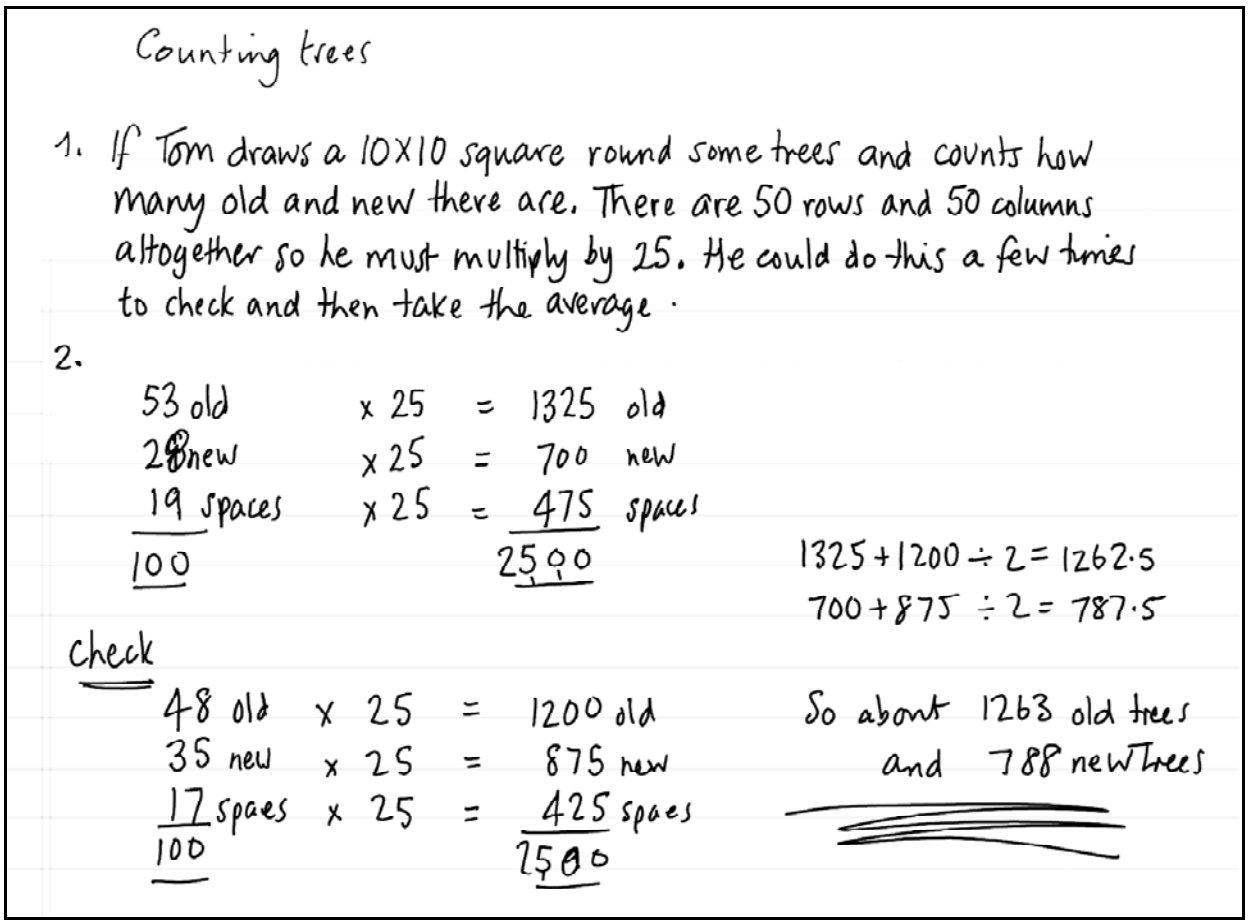


2 columns has 21 young trees
55 old

50 columns is approx
50 ÷ 2 = 25
25 × 21 = amount of young trees = 525
25 × 55 = amount of old trees = 1,375
rounded up

young 530
old 1,380

Sample response: Amber



Counting trees

1. If Tom draws a 10X10 square round some trees and counts how many old and new there are. There are 50 rows and 50 columns altogether so he must multiply by 25. He could do this a few times to check and then take the average.

2.

53 old	x 25	= 1325	old
28 new	x 25	= 700	new
19 spaces	x 25	= 475	spaces
100		2500	

1325 + 1200 ÷ 2 = 1262.5
700 + 875 ÷ 2 = 787.5

Check

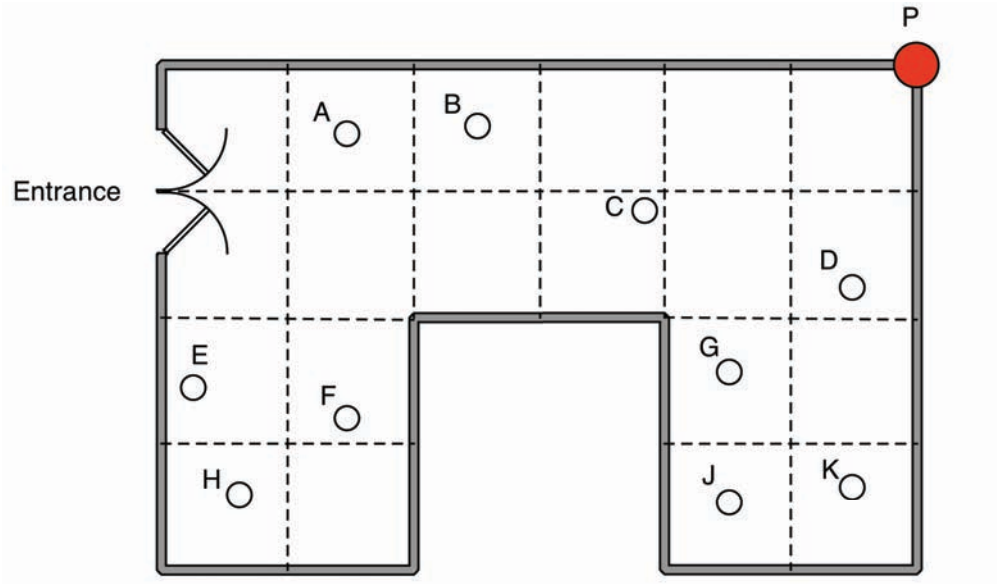
48 old	x 25	= 1200	old
35 new	x 25	= 875	new
17 spaces	x 25	= 425	spaces
100		2500	

So about 1263 old trees and 788 new trees

Security Camera

A shop owner wants to prevent shoplifting.
He decides to install a security camera on the ceiling of his shop.
The camera can turn right round through 360°.
The shop owner places the camera at point P, in the corner of the shop.
The plan below shows ten people are standing in the shop.

Plan view of the shop



1. Which people cannot be seen by the camera at P?

2. The shopkeeper says that "15% of the shop is hidden from the camera"
   Show clearly that he is right.

3. (a) Show the best place for the camera, so that the it can see as much of the shop as possible.

   (b) Explain how you know that this is the best place for the camera.

Sample response: Max

1. E, F and H cannot be seen by the camera.

2.

3a. The exact middle of the shop would be the place where it could see the most amount of people.

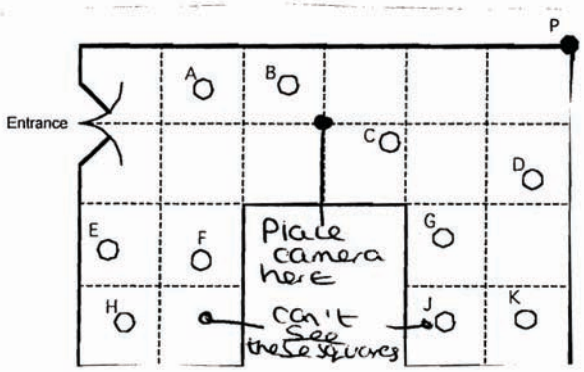
3b. Because the middle of the shop will grant the camera a larger vision of the so shop.

Sample response: *Ellie*

1. F + H

2. This is true because if there are 20 squared areas to make up the shop and 3 cannot be seen by the camera then that means the 3 squared areas would have to equal 15%. They do because if 15% of the room = 100% then to ge from 10 to 100 you divide by 10 and if you get 5 to 100 you divide by 2 and then by 10. adda them together and you'll get 15%.

3a+b I think the best place for the camera is in the centre of the room because it only can't see two squares.



Sample response: Simon

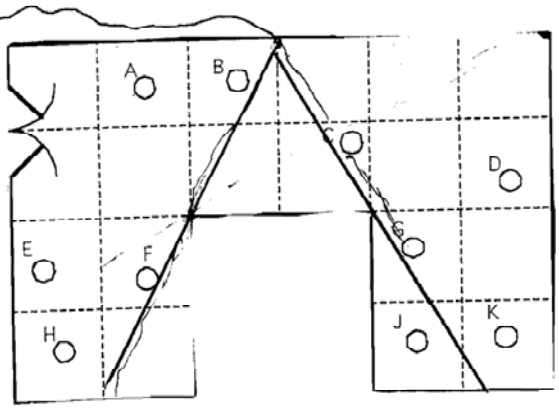
1. F + H

2. because 3 squares a hidden from the camera 1 square is 5% so 3 squares are 15%

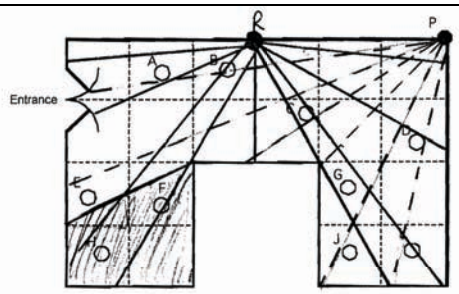
3.

a Here is the best place

b it can see all the carro almost every where



Sample response: *Rhianna*



1. He cannot see F + H.

2. There are 20 squares. 3 squares are hidden from the camera.
Each square represents 5%.
3 x 5% = 15%.
This proves 15% of the shop is hidden

3.

a) ● = R
My camera

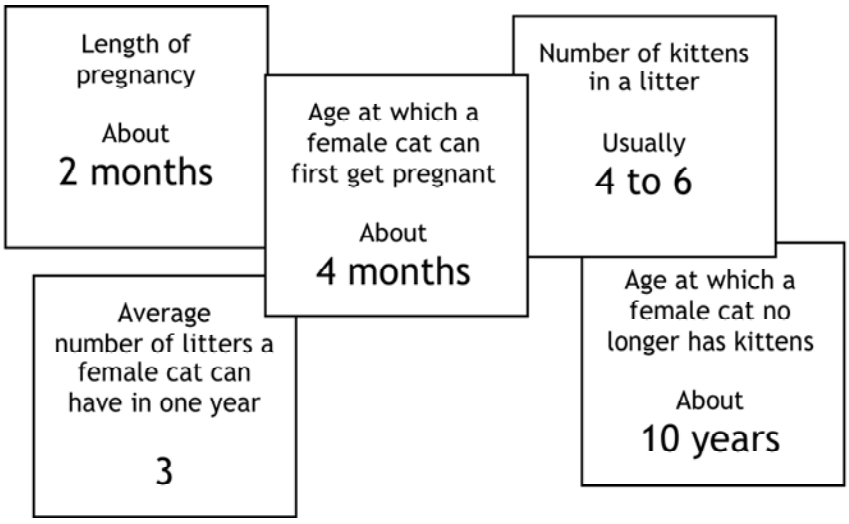
5% is hidden on one half.
5% is hidden on the other half.
This way only 10% is hidden + that space could be used for a till / trolleys.

B) I know this is the best place because it has a full view of all around the shop it can go ↺.

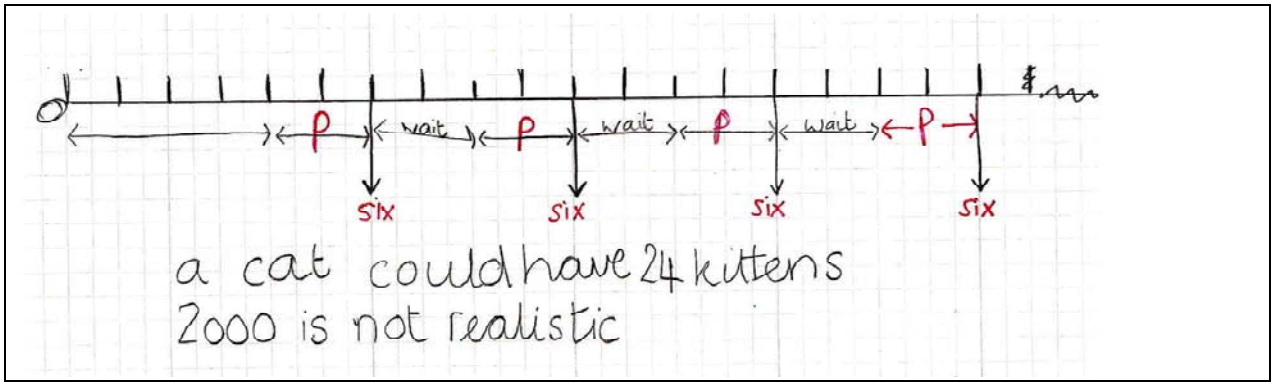
Cats and kittens

Here is a poster published by an organisation that looks after stray cats.

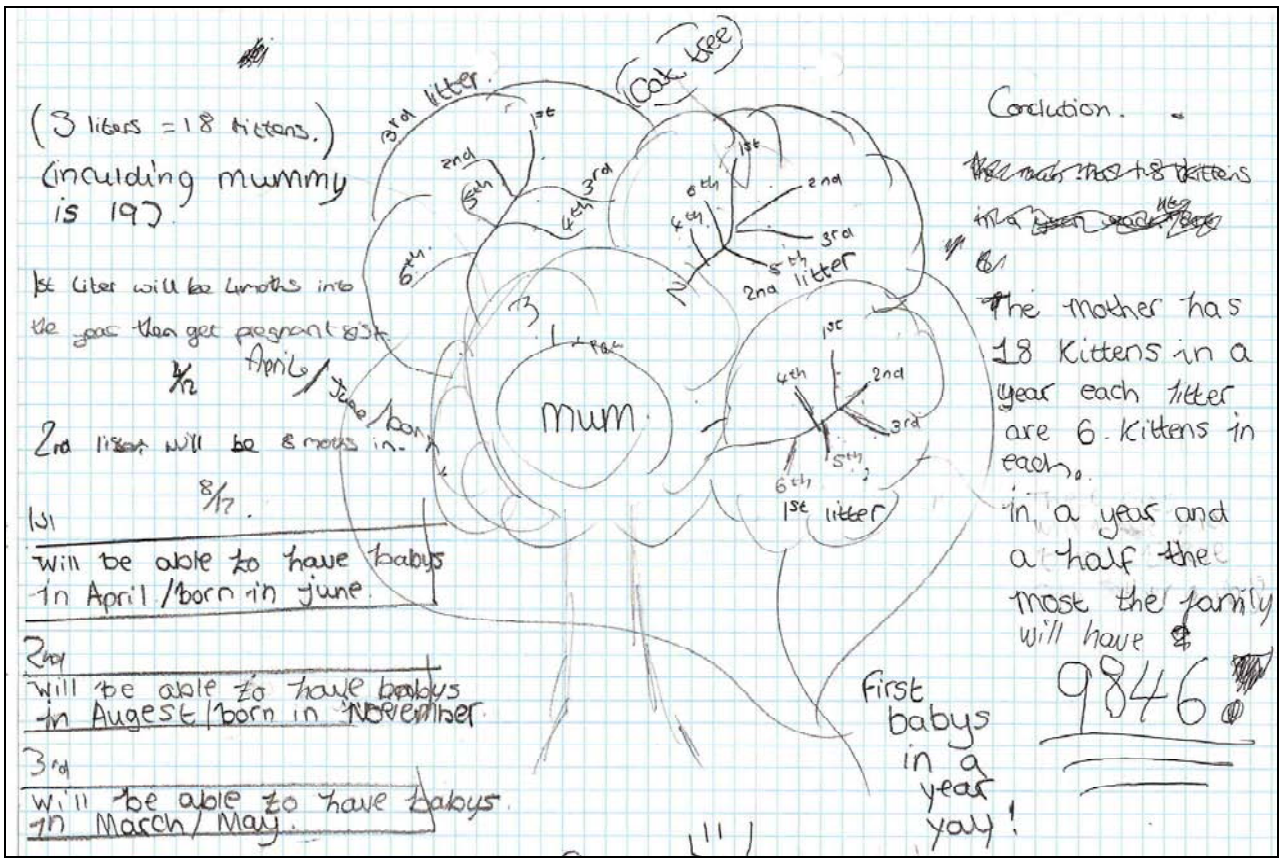
Work out whether this number of descendants is realistic.
Here are some facts that you will need:



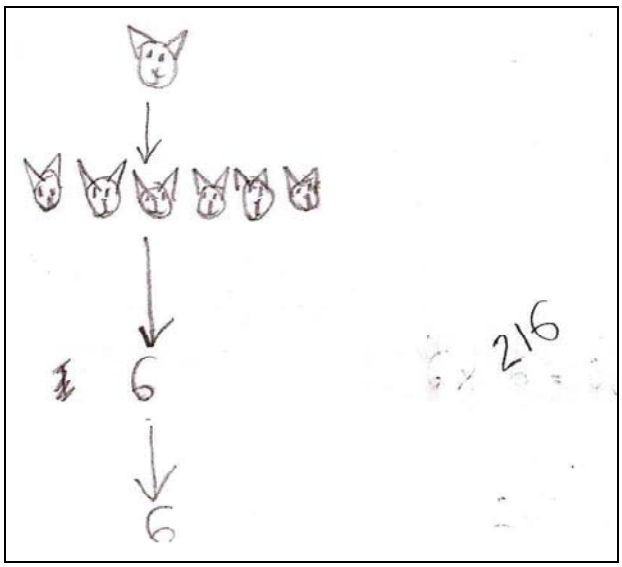
Sample response: Alice



Sample response: Ben

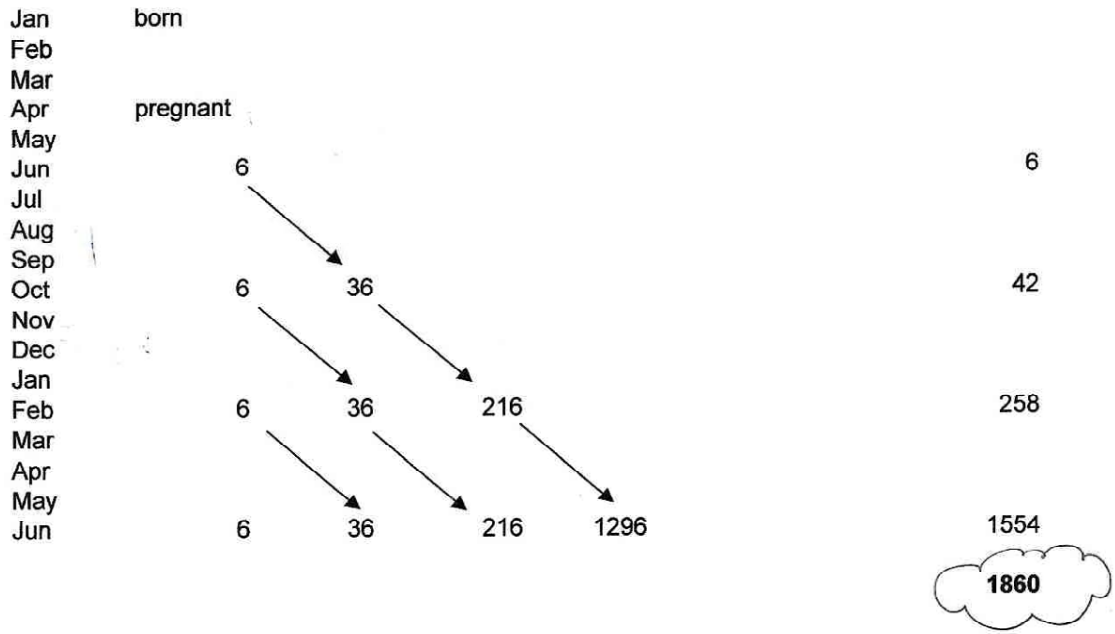


Sample response: Wayne



Sample response: Sally and Janet

Two pupils worked on this task, discussing and sharing their methods.
They used a spreadsheet.



We think 2000 is a bit much in 18 months because even if each litter was 6 and nothing dies there would be 1860 though that rounds to 2000 so maybe its OK. The cat people want owners to have their cats newtured so that they use the bigger number so that people say that is a lot of cats and rush to the vets.

3 Improving pupils' responses through questioning

Counting Trees

Sample response: Laura

Laura attempts to estimate the number of old and new trees by multiplying the number along each side of the whole diagram and then halving. She does not account for gaps nor does she realise that there are an unequal number of trees of each kind.

What questions could you ask Laura that would help her improve her response?

Sample response: Jenny

Jenny realises that sampling is needed, but she multiplies the number of young trees and old trees in the left hand column by the number of trees in the bottom row. She ignores the columns with no trees in the bottom row, so her method underestimates the total number of trees. She does, however, take account of the different numbers of old and new trees.

What questions could you ask Jenny that would help her improve her response?

Sample response: Woody

Woody uses a sample of two columns and counts the number of old and young trees. He then multiplies by 25 (half of 50 columns) to find an estimate of the total number.

What questions could you ask Woody that would help him improve his response?

Sample response: Amber

Amber chooses a representative sample and carries through her work to get a reasonable answer. She correctly uses proportional reasoning. She checks her work as she goes along by counting the gaps in the trees. Her work is clear and easy to follow.

What questions could you ask Amber that would help her improve her response?

Security Camera

Sample response: Max

Max realises that F and H cannot be seen, but incorrectly thinks that E cannot be seen. He does not show any work to justify his thinking and his further statements are incorrect.
Laura attempts to estimate the number of old and new trees by multiplying the number along each side of the whole diagram and then halving. She does not account for gaps nor does she realise that there are an unequal number of trees of each kind.

What questions could you ask Max that would help him improve his response?

Sample response: Ellie

Ellie does not show any sightlines to justify her answers. However, she correctly states that F and H cannot be seen and that 3 squares cannot be seen. However, she may be thinking of whole squares rather than areas,. Her justification for the 15% is incomplete and poorly explained. She seems to have some understanding that 5% is one twentieth and 10% is one tenth.

What questions could you ask Ellie that would help her improve her response?

Sample response: Simon

Simon correctly states that F and H cannot be seen and that 3 squares = 15% of the area cannot be seen. However, it is possible that he thinks that 3 whole squares are hidden from the camera. He investigates the best place for the camera, and shows that the centre of a side is good but he does not investigate further. No calculations are shown.

What questions could you ask Simon that would help him improve his response?

Sample response: Rhianna

Rhianna correctly shows that F and H cannot be seen and that 3 squares = 15% of the area cannot be seen. She investigates the best place for the camera, and shows that the centre of a side is good. Rhianna clearly shows diagrams with sightlines and calculations that justify her findings.

What questions could you ask Rhianna that would help her improve her response?

Cats and Kittens

Sample response: Alice

Alice chose to represent the task using a timeline. She has only considered the number of kittens from the original cat. The computation required is accurate.

What questions could you ask Alice that would help her improve her response?

Sample response: Ben

Ben has decided to draw a 'cat tree', and tries to control for time (with some errors). The communication is reasonably clear, allowing the reader to follow the argument, but the value of 9846 is not explained and does not follow from the reasoning, since, again, only the kittens from the original cat are considered. The number of kittens per litter is made explicit.

What questions could you ask Ben that would help him improve his response?

Sample response: Wayne

Woody appears to favour a minimalist approach! He starts with what would be a time consuming pictorial representation which he then abandons in favour of a numerical representation.

What questions could you ask Wayne that would help him improve his response?

Sample response: Sally and Janet

Sally and Janet used a spreadsheet to control for both time and multiplication and their method is clear and effective.

What questions could you ask Sally and Janet that would help them improve their response?

4 Questioning that will help improve responses

Use these generic questions to help plan specific questions to help the pupils' adapt their responses.

Representing	• How can we get started on this problem? • What maths might be useful here? • What sort of diagram might be helpful? • Can we invent a simple notation for this? • How can we simplify this problem? • What is known and what is unknown? • What assumptions might we make?
Reasoning	• Where have we seen something like this before? • What is fixed here, and what can we change? • What is the same and what is different here? • What would happen if I changed this.. to this...? • Is this approach going anywhere? • What will you do when you get that answer? • This is just a special case of ... what? • Can you form any hypotheses? • Can you think of any counterexamples? • What mistakes have we made? • Can you suggest a different way of doing this? • What conclusions can you make from this data? • How can we check this calculation without doing it all again? • What is a sensible way to record this?
Interpreting and evaluating	• How can you best display your data? • Is it better to use this type of chart or that one? Why? • What patterns can you see in this data? • What reasons might there be for these patterns? • Can you give me a convincing argument for that statement? • Do you think that answer is reasonable? Why? • How can you be 100% sure that is true? Convince me! • What do you think of Anne's argument? Why? • Which method might be best to use here? Why?
Communicate and reflect	• What method did you use? • What other methods have you considered? • Which of your methods was the best? Why? • Which method was the quickest? • Where have you seen a problem like this before? • What methods did you use last time? Would they have worked here? • What helpful strategies have we learned for next time?

5 The effects of feedback on pupils' learning

Read the following two extracts from Black and Wiliam (1998) and respond to the questions that follow:

The dangers of giving marks, levels, rewards and rankings

"Where the classroom culture focuses on rewards, 'gold stars', grades or place-in-the-class ranking, then pupils look for the ways to obtain the best marks rather than at the needs of their learning which these marks ought to reflect. One reported consequence is that where they have any choice, pupils avoid difficult tasks. They also spend time and energy looking for clues to the 'right answer'. Many are reluctant to ask questions out of fear of failure. Pupils who encounter difficulties and poor results are led to believe that they lack ability, and this belief leads them to attribute their difficulties to a defect in themselves about which they cannot do a great deal. So they 'retire hurt', avoid investing effort in learning which could only lead to disappointment, and try to build up their self-esteem in other ways. Whilst the high-achievers can do well in such a culture, the overall result is to enhance the frequency and the extent of under-achievement."

- What are the implications of this for your practice?
- What would happen if you stopped giving marks or levels on pupils' work?
- Why are so many teachers resistant to making this change?

The advantages of giving clear, specific, content-focused feedback

"What is needed is a culture of success, backed by a belief that all can achieve. Formative assessment can be a powerful weapon here if it is communicated in the right way. Whilst it can help all pupils, it gives particularly good results with low achievers where it concentrates on specific problems with their work, and gives them both a clear understanding of what is wrong and achievable targets for putting it right. Pupils can accept and work with such messages, provided that they are not clouded by overtones about ability, competition and comparison with others. In summary, the message can be stated as follows:

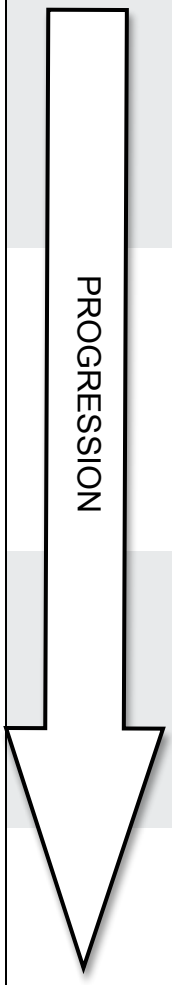
Feedback to any pupil should be about the particular qualities of his or her work, with advice on what he or she can do to improve, and should avoid comparisons with other pupils."

- What are the implications of this for your practice?
- Does this kind of feedback necessarily take much longer to give?

Black, P., & Wiliam, D. (1998). *Inside the black box : raising standards through classroom assessment*. London: King's College London School of Education 1998.

6 Progression steps in the Key Processes

Counting Trees



	Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
	Chooses a method, but this may not involve sampling. E.g. Counts all trees or multiplies the number of trees in a row by the number in a column.	Follows chosen method, possibly making errors. E.g. Does not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due to method and errors.	Communicates work adequately but with omissions.
	Chooses a sampling method but this is unrepresentative or too small. E.g. tries to count the trees in first row and multiplies by the number of rows.	Follows chosen method, mostly accurately. E.g. May not account for different numbers of old and young trees or that there are gaps.	Estimates number of new and old trees, but answer given is unreasonable due mainly to the method.	Communicates reasoning and results adequately, but with omissions.
	Chooses a reasonable sampling method.	Follows chosen method, mostly accurately.	Estimates a reasonable number of old and new trees in the plantation. The reasonableness of the estimate is not checked. E.g. by repeating with a different sample.	Explains what they are doing but explanation may lack detail.
	Chooses an appropriate sampling technique.	Follows chosen method accurately. Uses a proportional argument correctly.	Deduces a reasonable number of old and new trees in the plantation. There is some evidence of checking the estimate. E.g. Considers a different sampling method.	Communicates reasoning clearly and fully.

6 Progression steps in the Key Processes

Security Camera

	Representing	Analysing	Interpreting and evaluating	Communicating
PROGRESSION	Sight lines are not drawn and there is little evidence of the student selecting other suitable mathematical methods	The student may realise that the camera at P cannot see some people, but gives little justification	The student may consider another place for the camera but this is not optimal and the justification is incorrect	The work is communicated adequately, but there are gaps and/or omissions.
	Sight lines may not be drawn but there is some evidence of the student selecting other suitable mathematical methods	Some partial correct analysis with some justification, but there are errors. The student may realise that F and H cannot be seen but may also think that E cannot be seen	Has some idea where a better place for the camera is e.g. 'above B on ceiling'	The work is communicated clearly and the reasoning may be followed.
	Sight lines are used in some parts of the work.	Gives a mostly correct justification as to why 15% of the room may not be seen from P. May think that 3 whole squares rather than an area equal to 3 squares cannot be seen from P.	Correctly finds a better place for the camera, but justification may be incomplete	The work is communicated clearly and the reasoning may be easily followed.
	Selects and uses sight lines in all parts of the work to get accurate answers	Finds the correct percentage of the shop that is hidden.	Successfully compares areas from various viewpoints and finds an optimal point.	Explains work clearly and considers alternative solutions

6 Progression steps in the Key Processes

Cats and Kittens

	Representing	Analysing	Interpreting and evaluating	Communicating and reflecting
PROGRESSION	Draws a simple diagram or Draws a timeline with some key events shown sequentially	Finds the number of kittens that would exist if each cat had only one litter	Relates their findings to the original problem, e.g. by stating whether 2000 descendants is or is not realistic	Presents work in such a way that it is possible to determine which is the original cat, and how many kittens are within each litter
	Draws a simple diagram and shows or implies multiplication is an appropriate mathematical tool *or* Draws a timeline with some key events shown sequentially, considering more than just the offspring of the first cat	Uses multiplication to find the number of kittens that would exist if each cat had only one litter and recognises the need to count all those descendants	Makes explicit the assumption about the number of kittens per litter, e.g. 'Each litter is 6 kittens'	Shows methods so that someone else can follow their reasoning reasonably well
	The chosen method represents both multiplication and time for the original kitten even if not all her descendants are represented	Recognises that most cats, in the time available, can have more than one litter	Qualifies assumptions about the number of kittens per litter. E.g. 'I used 6 – that gives the biggest number of cats'	Throughout the task there is clear, effective and concise communication that builds to a solution, even if partial
	The chosen method represents both multiplication and time for the original kitten and all her descendants	Uses an effective method to work towards a credible solution that takes into account the wide range of factors within the task	Makes explicit further assumptions. E.g. No cats die or that cats become pregnant as soon as physically possible	Throughout the task there is clear, effective and concise communication with evidence of reflection. E.g. The number of kittens per litter affects the outcome significantly

7 Suggestions for further reading

Black, P., & Wiliam, D. (1998). *Inside the black box: raising standards through classroom assessment*. King's College London School of Education.
Now published by GL Assessment: http://shop.gl-assessment.co.uk

This short booklet offers a summary of the extensive research literature into formative assessment. It shows that there is clear evidence that improving formative assessment raises standards, and offers evidence showing how formative assessment may be improved. This booklet is essential reading for all teachers.

Black, P., & Harrison, C. (2002). *Working inside the black box: Assessment for learning in the classroom*. King's College London School of Education.
Now published by GL Assessment: http://shop.gl-assessment.co.uk

In this booklet, the authors describe a project with teachers in which they studied practical ways of implementing formative assessment strategies and the effect this had on learning. The section on feedback and marking (pages 8-9) are particularly relevant to this module.

Black, P., Harrison, C., Lee, C., Marshall, B., & Wiliam, D. (2003). *Assessment for learning: Putting it into practice*. Buckingham: Open University Press.

This book gives a fuller account of the earlier booklets *Inside the black box* and *Working inside the black box*. It discusses four types of action: questioning, feedback by marking, peer- and self-assessment and the formative use of summative tests. The section on feedback and marking (pages 42-49) is particularly relevant to this module, while the section on peer and self-assessment (pp 49-53) is relevant for the next CPD module.

Hodgen, J., & Wiliam, D. (2006). *Mathematics inside the black box*. King's College London School of Education. Now published by GL Assessment: http://shop.gl-assessment.co.uk

This booklet applies the above findings specifically to Mathematics. It considers some principles for Mathematics learning, choice of activities that promote challenge and dialogue, questioning and listening, peer discussion, feedback and marking, and self and peer assessment. This booklet is essential reading for all mathematics teachers.