Fostering and managing collaborative work
'How can I get pupils to stop talking and start discussing?'

There is overwhelming evidence that mathematical discussion is beneficial for learning when pupils engage with each others' reasoning. This module is intended to help you:

- consider the characteristics of an effective pupil-pupil discussion;
- explore techniques for promoting pupil-pupil discussion;
- discuss the teacher's role in managing pupil-pupil discussion.

This guide is intended for use alongside the *Bowland Maths DVD* or website, which include a short introductory video for each of the activities; longer videos of lessons and teacher discussions and links to all the handouts and ICT-based problems. Due to the nature of this particular module, it is helpful to work through it with a group of colleagues.

Introductory session	1 hour

- Experience a mathematical discussion
- Reflect on your discussion
- Observe a discussion lesson
- Discussing implications for teaching
- Plan a lesson using one of the problems.

Into the classroom	1 hour

- Introduce the problem and pupils think on their own
- Pupils share their ideas in pairs
- Discuss some helpful ways of working
- Pupils have another go at the problem
- Pupils share solution strategies

Follow-up session	1 hour

- Report and reflect on the lesson
- Consider your own role during collaborative work
- Devise strategies for pupils who struggle to communicate
- Plan some strategies to use in future lessons

Resources Needed	

Handout 1	Problems for discussion
Software	Treasure hunt (optional)
Handout 2	Recognising helpful and unhelpful talk
Handout 3	Ten ground rules for pupil-pupil discussion
Handout 4	Planning for pupil-pupil discussion
Handout 5	What is the teacher's role during discussion?
Handout 6	Notes on the problems
Handout 7	Further activities to promote speaking and listening
Handout 8	Suggested further reading

BOWLAND
MATHS
Professional development

Introductory session

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Introduction	**The importance of discussion**

We know from research that mathematical discussion is an **essential** component of thinking and reasoning. Yet, as many OfSTED reports confirm, collaborative discussion is rare among pupils in most mathematics classrooms.

This module is intended to help you:
- consider the characteristics of an effective pupil-pupil discussion;
- explore techniques for promoting pupil-pupil discussion;
- discuss the teacher's role in managing pupil-pupil discussion.

Activity 1	**Experience a mathematical discussion**	**10 minutes**

What are the characteristics of helpful and unhelpful classroom talk? Before discussing this question in detail, we suggest that you experience a mathematical discussion for yourself with a small group of colleagues.

Discuss the following problem together:

How many people can stand comfortably on a football pitch?

Alternatively, you may prefer to tackle one of the problems on 📄 **Handout 1**. These problems are similar to those found in the Bowland Case Studies.

You may like to compare your discussion with that held by three teachers: Marc, Eve and Angela.

Activity 2	Reflect on your discussion	10 minutes

Take some time to reflect on the experience you have just had.

- What roles did you and your colleagues play in the discussion?

Not all kinds of classroom talk are helpful for learning. Refer to characteristics of helpful and unhelpful talk on **Handout 2**.

- Which of these characteristics do you recognise in your own discussion?

- Was your discussion:
  Collective? How far did you really think together, or did you tend to follow independent lines of thought? Did someone 'take over'? Was someone a 'passenger'?
  Reciprocal? Did you listen to, share ideas with and consider the alternative views of everyone in the group?
  Cumulative? Did you build on each others' ideas to construct chains of coherent reasoning?
  Supportive? Did you feel able to share your ideas without fear of embarrassment of being wrong? Did anyone feel uncomfortable or threatened? If so, why?
  Purposeful? Did your discussion stay 'on task' or were you 'wandering'?

- Would you describe your talk as **Disputational**, **Cumulative** or **Exploratory**?

- What did you learn mathematically from this experience?

- What concepts, skills and problem solving strategies were being developed?

Activity 3	Observe a discussion lesson	20 minutes

The video clips show three teachers; Eve, Angela and Marc teaching with the three problems shown on **Handout 1**. We suggest that you watch Eve's lesson *(How many school teachers are there in the UK?)* first. Later, you may wish to come back and watch Angela and Marc's lessons.

Watch the video clip, and then consider the following issues, referring again to **Handout 2**:

- How does the teacher introduce the problem?
- What different approaches are being used by pupils?
- How does the teacher help pupils to discuss productively?
- Can you characterise the types of talk they are using?

Activity 4	Discuss implications for teaching	10 minutes
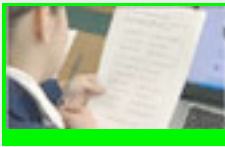
Pupils (and adults!) do not always discuss in helpful ways. Some are reluctant to talk at all, while others take over and dominate. Pupils may therefore need to be *taught how* to discuss. Some teachers have found it helpful to introduce a list of 'ground rules for discussion' into their classes. These ground rules should, in appropriate language, give explicit guidance to pupils on how to talk together profitably.

- Together in your group, prepare your own list of "Ground rules for discussion".
- Compare your list with that offered on 🔲 **Handout 3.**
- How could you encourage your pupils to follow these rules?
- Could you involve your pupils in drawing up such a list?

Activity 5	Plan a lesson using one of the problems (10 minutes)	10 minutes

Choose one problem from 🔲 **Handout 1** that would be appropriate for your class and plan a lesson.

- How will you organise the classroom and the resources needed?
- How will you group pupils? For example, will you deliberately pair pupils who will discuss well together? Will you need to keep some apart?
- How will you introduce the problem?
- How will you explain *how* you want pupils to discuss; which ground rules will you introduce?
- How will you manage the discussion? For example, will you give the pupils a chance to work on the problem individually, before moving them into pairs or groups?
- Will you hold a plenary discussion towards the end of the lesson?

You may like to watch the video of Eve planning her lesson using *Schoolteachers and dentists*.

🔲 **Handout 4** contains some general advice on planning for pupil-pupil discussion.

🔲 **Handout 6** gives some background notes on the problems and their relevance to the Bowland Case Studies.

This is the end of the *Introductory session*. After you have tried out your lesson with your own pupils, return for the *Follow-up session*.

Resources to support the lessons, and suggested lesson plans, can be found in the *Into the classroom* session.

Resources you will need:

Schoolteachers and dentists:	Mini whiteboards and felt pens.
Sharing office space:	1cm squared paper, mini whiteboards.
Treasure hunt:	Software, 1 computer per pair of pupils, 1 mm graph paper.

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The following suggestions describe one possible approach to using the problems with pupils. This may take one or two lessons, depending on the class.

Introduce the problem and pupils think on their own	5 minutes

Issue each pupil with a copy of just one of the problems. Introduce the problem to the class. In the case of Treasure Hunt, do a few moves with them on the interactive whiteboard so that they understand what to do.

Explain the purpose of the lesson and give pupils two minutes to think about the problem on their own.

> *The aim of today's lesson is to see how well you can work with a partner to solve a problem. But first I want you to spend two minutes reading the problem quietly on your own so that you have time to think about the problem before you start discussing it. Jot down your own ideas on paper.*

Pupils share their ideas in pairs	10 minutes

Explain that you now want them to work in pairs, taking it in turns to share their ideas.

> *Now I want you to work in pairs.*
> *Take turns at explaining your ideas for tackling the problem.*
> *Listen carefully to each other. If you don't understand what is said then challenge your partner to explain themselves more clearly.*
> *When you have both explained your ideas, then try to agree on the best approach together.*

Discuss some helpful ways of working	10 minutes

Give out copies of 🗐 **Handout 3** and make the point that people don't learn by simply agreeing or disagreeing, but by following these "ground rules". Ask a few pupils to say which ground rule they find most helpful.

Pupils have another go at the problem	25 minutes

Ask pupils to discuss the problem again using the ways of talking in the ground rules. As

Try to listen to pupils' discussions before intervening. Then join in the group asking them gently to describe, explain and interpret without judging their responses as 'good or bad'. Above all, do not do the thinking for pupils.

If possible, tape-record one 5 minute episode of a pupil-pupil discussion, for use in the follow-up meeting.

Pupils share solution strategies   10 minutes

Conclude the lesson by inviting pupils to share their reasoning with the whole class. Emphasise that you want to hear *reasoning*, not just their answers. As pupils present their ideas, ask other pupils to comment on the advantages and disadvantages of each approach.

Congratulate them on the ground rules they have used and ask them to consider where they still need to improve.

An optional activity: Pupils watch a video of other people working together

In order to help pupils reflect on the ground rules further you may like to ask them to tackle the problem:

> *How many people can comfortably stand on a football pitch?*

Show the video clip of teachers discussing the same problem (in the *Introductory session* for this module on the DVD/website). Stop the video and ask the class to discuss what was good about the discussion they saw and what could be improved. They might like to begin by commenting on the mathematics and then on the way in which the teachers discussed together. Refer again to **Handout 3**.

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Activity 1	Report and reflect on the lesson	20 minutes

Now you have taught the lesson, it is time to reflect on what happened.

Compare the different ways in which you planned the lessons.

How did you:
- Organise the room and introduce the task?
- Orchestrate and sustain phases of the work? (E.g. "Think, pair, share")
- Organise and share ideas as a whole class?

Give examples of helpful and unhelpful discussions that were evident.

- Did pupils listen to and build on each others' reasoning?

Play any recorded extracts of pupil-pupil talk from your lesson and discuss the type of talk this illustrates.

- Can you think of occasions when your own intervention was helpful? When was it unhelpful?

Activity 2	Consider your own role during collaborative work	10 minutes

While pupils are discussing, teachers often find it difficult to know what to do. When and how should they intervene? The character and content of pupil-pupil talk can change dramatically in nature when the teacher joins in!

- What do you do while pupils talk together?
- How do you decide when to intervene and when to leave them struggling?
- What helpful and unhelpful things do you say or do?
- Do you try to 'put pupils right' as soon as you hear them going off track, or do you resist this urge? Why?
- Consider the advice on 🗂 **Handout 5**. Which pieces of advice do you find most pertinent?

Activity 3	Devise strategies for pupils who struggle to communicate	10 minutes

For pupils who struggle to communicate, it is sometimes easier to begin by developing their speaking and listening skills through activities that require *describing* rather than *problem solving.*
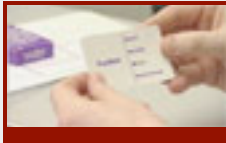
In the video clip, Eve, Angela and Marc try out two activities designed to promote speaking and listening in mathematics lessons. These ideas are explained more fully on **Handout 7**.

Watch the video clip.

- What types of talk do these activities generate?
- What other activities could you use to help your pupils improve their speaking and listening?

Activity 4	Plan some strategies for future lessons	10 minutes

Plan some ways of using what you have learned in this PD module to other mathematics lessons that you teach.

- Choose one topic that you plan to teach next week.
- Think of some ways in which you might build in a mathematical discussion of the important ideas in that topic

Further reading

See **Handout 8** for a list of references.

1   Problems for discussion

Estimating and interpreting: **Schoolteachers and dentists**

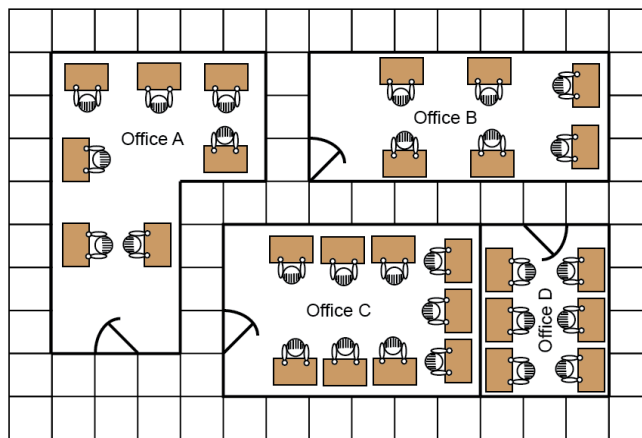There are about 60 million people in the UK.

- About how many school teachers are there?
- About how many dentists are there?

Estimate some other facts and check them out!

Modelling and explaining: **Sharing office space**

This drawing shows four offices in a factory. The workers complain that some offices are more crowded than others. How could the workers be rearranged with the minimum of fuss, so that the offices are equally crowded?

Solving logic puzzles: **Treasure hunt**

Type coordinates into the yellow spaces to find the treasure, using the clues given by the pirate.

What advice would you give someone to help them find the treasure with the least amount of digging?

Treasure Hunt

Go North and go East

The treasure is buried on the 100x100 grid. Try to find the treasure with the least amount of digging! Enter the coordinates of your next dig:

(20 , 30)   Dig   Digs: 1

2 Recognising helpful and unhelpful talk

What types of talk engages pupils, develops understanding and promotes deeper thinking? Robin Alexander (2006)[1] identified the following five principles of helpful classroom talk - which he terms *dialogic*.

Dialogic talk is:

- *Collective*: teachers and children address learning tasks together, as a group or as a class, rather than in isolation
- *Reciprocal*: teachers and children listen to each other, share ideas and consider alternative viewpoints
- *Cumulative*: teachers and children build on their own and each others' ideas and chain them into coherent lines of thinking and enquiry
- *Supportive*: children articulate their ideas freely, without fear of embarrassment over 'wrong' answers and they help each other to reach common understandings
- *Purposeful*: teachers plan and facilitate dialogic teaching with particular educational goals in view

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Neil Mercer (1995, 2000)[2] identifies the following three types of pupil-pupil talk. It is the third type, exploratory talk, that is most helpful fo learning:

Cumulative talk	Speakers build positively, but uncritically on what each other has said. This is typically characterised by repetitions, confirmations and elaborations.
Disputational talk	This consists of disagreement and individualised decision making. It is characterised by short exchanges consisting of assertions and counter-assertions.
Exploratory talk	Speakers work on and elaborate each other's reasoning in a collaborative, rather than competitive atmosphere. Exploratory talk enables reasoning to become audible and knowledge becomes publicly accountable. It is characterised by critical and constructive exchanges. Challenges are justified and alternative ideas are offered.

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[1] Alexander, R. (2006). Towards Dialogic Teaching: Rethinking Classroom Talk (3 ed.). Thirsk: Dialogos.

[2] Mercer, N. (1995). The guided construction of knowledge. Clevedon, Philadelphia, Adelaide. Mercer, N. (2000). Words and Minds. London: Routledge.

3 Ten ground rules for pupil-pupil discussion

Here are some suggested 'ground rules' for **pupils** to use as they work in groups.
These could be displayed and reinforced over time.
Maybe you could involve you class in drawing up a similar list.

1.	**Give everyone in your group a chance to speak**	"Lets take it in turns to say what we think". "Claire, you haven't said anything yet."
2.	**Listen to what people say**	"Don't interrupt - let Sam finish". "I think Sam means that ...."
3.	**Check that everyone else listens**	"What did Sue just say?." "I just made a deliberate mistake - did you spot it?
4.	**Try to understand what is said**	"I don't understand. Can you repeat that?" "Can you *show* me what you mean?"
5.	**Build on what others have said**	"I agree with that because ..." "Yes and I also think that ...."
6.	**Demand good explanations**	"Why do you say that?" "Go on ... convinced me."
7.	**Challenge what is said**	"That cannot be right, because..." "This explanation isn't good enough yet."
8.	**Treat opinions with respect**	"That is an interesting point." "We all make mistakes!"
9.	**Share responsibility**	"Let's make sure that we are all able to report this back to the whole class."
10.	**Reach agreement**	"We've got the general idea, but we need to agree on how we will present it."

4   Planning for pupil-pupil discussion

Plan to offer the task in a form that will encourage collaboration	Prepare shared tasks in a *form* that will encourage discussion. For example: • Provide resources *to share* (e.g. one copy between three) and ask for outputs that are jointly produced. • Provide *big* resources so that reasoning may be visible and shared, such as large sheets of paper, felt-tipped pens or 'mini-whiteboards'. • Require *joint outcomes*: e.g. a poster or a report. Make pupils share responsibility for this.
Plan how you will arrange the room	Arrange tables and chairs so that pupils are facing each other while working together. When computers are used, then pair two pupils to a computer and give them space and resources to record their joint thinking (e.g. using mini-whiteboards). Encourage turn taking when using the computer.
Plan how you will group pupils	Most pupils are more able to discuss in smaller groups than larger ones: pairs or threes is often most effective. Some teachers find a *snowball* approach helpful: • Pupils first tackle the task individually. They have time to think before they are asked to discuss. • Pairs are then formed and pupils are asked to try and reach agreement. • Pairs then join together so that a broader consensus might be reached. • Groups of four then report back to the whole class in a plenary discussion.
Plan how you will introduce the purpose of discussing	Plan your introduction to pre-empt the questions: • "Why do you want us to discuss?" • "What do you want us to discuss?" For example: *This lesson is not about 'me showing you a method and then you using it'. No, I want to see if you can find your own methods. There is more than one way of doing this! I want you to discuss your own ideas for starting on this problem.*
Plan how you will establish ground rules	Introduce ground rules for pupils such as those outlined on **Handout 3**. Such behaviours are not established overnight, but over a long time through consistent reinforcement.
Plan how you will end the discussion	Most teachers ask pupils to report back on their discussion in some way. All pupils should be encouraged to prepare for this. Try not to pass judgments on their responses while they do this or this may influence subsequent contributions. (See **Handout 5**)

5   What is the teacher's role during discussion?

Make the purpose of the task clear	Explain what the task is and how they should work on it. Also, explain why they should work in this way. 'Don't rush, take your time. The answers are not the focus here. It's the *reasons* for those answers that are important. You don't have to finish, but you do have to be able to explain something to the rest of the class.'
Keep reinforcing the 'ground rules'	Try to ensure that pupils remember the ground rules that were discussed at the beginning. Encourage pupils to develop a responsibility for each other's understanding. 'I will pick one of you to explain this to the whole class later – so make sure all of you understand it'.
Listen before intervening	When approaching a group, stand back and listen to the discussion before intervening. It is all too easy to interrupt a group with a predetermined agenda, diverting their attention from the ideas they are discussing. This is not only annoying and disruptive (for the group), it also prevents pupils from concentrating.
Join in, don't judge	Try to join in as an equal member of the group rather than as an authority figure. When teachers adopt judgmental roles, pupils tend to try to 'guess what's in the teacher's head' rather than try to think for themselves: 'Do you want us to say what we think, or what we think you want us to say?'
Ask pupils to describe, explain and interpret	The purpose of an intervention is to increase the depth of reflective thought. Challenge pupils to describe what they are doing (quite easy), to interpret something ('Can you say what that means?') or to explain something ('Can you show us why you say that?).
Do not do the thinking for pupils.	Many pupils are experts at making their teachers do the work! They know that if they 'play dumb' long enough, then the teacher will eventually take over. Try not to fall for this. If a pupil says that he or she cannot explain something, ask another pupil in the group to explain, or ask the pupil to choose some part of the problem that she can explain. Don't let them off the hook! When a pupil asks the teacher a question, don't answer it (at least straight away). Ask someone else in the group to do so.
Don't be afraid of leaving discussions unresolved.	Some teachers like to resolve discussions before they leave the group. When the teacher leads the group to the answer, then leaves, the discussion has ended. Pupils are left with nothing to think about, or they go on to a different problem. It is often better to reawaken interest with a further interesting question that builds on the discussion and then leave the group to discuss it alone. Return some minutes later to find out what has been decided

6   Notes on the problems

Estimating and interpreting: **Schoolteachers and dentists**

Links to Case Studies

This task offers an unstructured estimation problem. Such problems are intended to create confidence in pupils' own ability to use knowledge that they already have.
This problem is similar to other problems within the Case Studies, most notably those that are contained within the Case Study "**You reckon?"**

Sample solutions

How many schoolteachers are there in the UK?

The solution will be dependent on the assumptions made and the line of reasoning taken. The solutions below illustrate just one approach.

We can make the following assumptions:
- About one fifth of the total population is at school. (If life span is approx 75 years and 14 of these are at school).
- Assume an average of 25 children per class.
- Assume all teachers are working full-time

Then there will be about $\dfrac{60,000,000}{5 \times 25}$ = 480,000 teachers.

Answer: about half a million teachers are required. In fact there are about 441,000 full time teachers in mainstream education (Source: www.statistics.gov.uk)

How many dentists are there in the UK?

We can make the following assumptions:
- Each member of population sees dentist on average twice per year.
- Each consultation lasts about 20 minutes
- Dentist sees patients for about 6 hours per day
- Dentist works for 45 weeks per year.

Then, dentist can have 45 x 5 x 6 x 3 = 4,050 consultations per year.
Number of consultations required = 120 million per year
120 million / 4,050 = 30 thousand dentists

Answer: about 30 thousand dentists are required. In fact there are about 31,000 registered dentists in the UK (Source BBC.co.uk)

6   Notes on the problems (continued)

Modelling and explaining:	Sharing office space

Links to Case Studies

This is similar to other problems within the Case Studies, where pupils are expected to model a situation (e.g. by creating a measure for something). Those most closely related are:
- **Water availability**: Create a fair way to distribute water.
- **My music**: Create a measure of tempo.
- **Keeping the pizza hot**: Model the cooling of a pizza

Sample solution

We first need to explain what is meant by 'more crowded'. One possible measure is the number of people per unit area.

At present:
- Office A has 7 people in 27 square units
- Office B has 6 people in 21 square units
- Office C has 9 people in 24 square units
- Office D has 6 people in 12 square units.

This is a total of 28 people in 84 square units.

If the space was shared equally, each office worker would get 3 square units of space. We could do this if we allocated people to offices so that:

- Office A has 9 people
- Office B has 7 people
- Office C has 8 people
- Office D has 4 people

One way of achieving this with least fuss would be to move 2 people from office D to office A and one person from office C to office B.

Of course this solution does not take into account the space needed for the doors!

6   Notes on the problems (continued)

Solving logic puzzles: Treasure hint

Links to Case Studies

There are a number of Case Studies that contain logic problems that are quite abstract in nature. These are often situated in the context of computer games.

Examples are:
- **Alien invasion**
- **PointZero**
- **Explorers**

Sample solution

The treasure may be hidden in 100x100 = 10,000 different places, so it will seem surprising to many pupils that it can always be located in no more than seven digs, provided that each dig is located in the centre (or very near to the centre) of the remaining region. In this way, the number of possible remaining hiding places is reduced by about one quarter each time. This strategy is shown in the following example:

Coordinates of dig	Resulting clue
(50, 50)	"Go East and go South"
(75, 25)	"Go East and go South"
(88, 12)	"Go West and go South"
(82, 6)	"Go East and go North"
(85, 9)	"Go West and go South"
(84, 8)	"Go West and go South"
(83, 7)	Arrh! Ye have found the treasure!

This strategy is, however, by no means obvious to many pupils and it will be interesting to see which strategies they can come up with unaided!

7   Further activities to promote speaking and listening

The following activities are designed for pupils who find it difficult to take it in turns to speak and listen. They do not generate mathematical discussions as such but rather offer a structured format in which pupils can speak for extended periods without interruption.

Back-to-back describing and making.

Ask pupils to sit in pairs, back to back. Give one pupil a picture or a physical object. The pupil then carefully describes the picture or object so that his or her partner can recreate it, without looking. For example:

- One pupil describes a picture (for example, a geometrical design such as the one shown here), the other tries to draw it.
- One pupil describes a 3D construction made from "Lego" or multilink cubes. The other tries to make the same construction using an identical set of material.
- One pupil gives directions to a place, the other draws the route on a map.
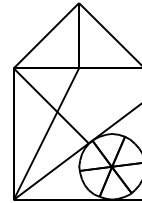
After each activity ask pairs to compare their product with the original. What was good about the 'describers' instructions? How might they have been changed to create a more accurate reproduction?

Defining words

Arrange pupils into pairs. Give one pupil in each pair a set of ten mathematical words. (e.g. quadrilateral, parallel, obtuse).
This pupil must try to communicate these words so that their partner can write them down, without using the words themselves, without drawing and without gesticulating in any way. They can only do this by explaining what the word means.

One variation of this game involves banning certain words from being used in the explanation. For example, the pupil may try to communicate the word 'square' without using the words 'four', 'straight sides' and so on.

Sets of cards with suitable mathematical terms (and banned words) are sold under the title "Fourbidden". These may be obtained from the Association of Teachers of Mathematics at http://www.atm.org.uk/buyonline/products/act013.html

8   Suggested further reading:

How can we be sure that the classroom encourages talk for learning? Here is what research shows.
Alexander R (2008) *Towards Dialogic Teaching: rethinking classroom talk* (Dialogos Cambridge
http://www.robinalexander.org.uk/docs/TDTform.pdf

What are the characteristics of talk for learning?
Mercer, N. (2000). *Words and Minds*. London: Routledge.

Improving learning in mathematics – through collaboration
Swan, M. *Improving Learning in Mathematics*, The Standards Unit.
http://www.ncetm.org.uk/files/224/improving_learning_in_mathematicsi.pdf

A research study into the design of collaborative classroom activities
Swan, M. (2006*). Collaborative Learning in Mathematics: A Challenge to our Beliefs and Practices*. London: National Institute for Advanced and Continuing Education (NIACE); National Research and Development Centre for Adult Literacy and Numeracy (NRDC).

Making your interactive whiteboard really interactive.
Tanner H & Jones S (2007) How interactive is your whiteboard?, *Mathematics Teaching #200,* ATM, Derby
http://www.atm.org.uk/mt/archive/mt200files/ATM-MT200-37-41-mo.pdf

This article is about practice in a primary school but has a lot to say to secondary school teachers.
Williamson V (2007) Group and individual work, *Mathematics Teaching #195,* ATM, Derby
http://www.atm.org.uk/mt/archive/mt195files/ATM-MT195-42-45-mo.pdf